Quantum Heat Engine Simulated on Superconducting Qubits

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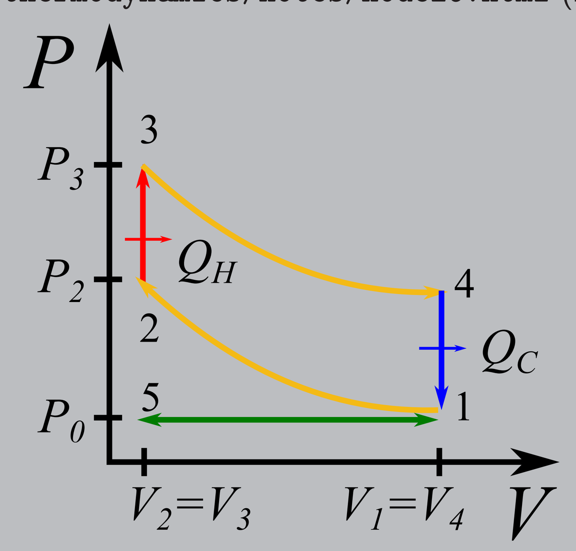
Classical vs. Quantum Heat Engines

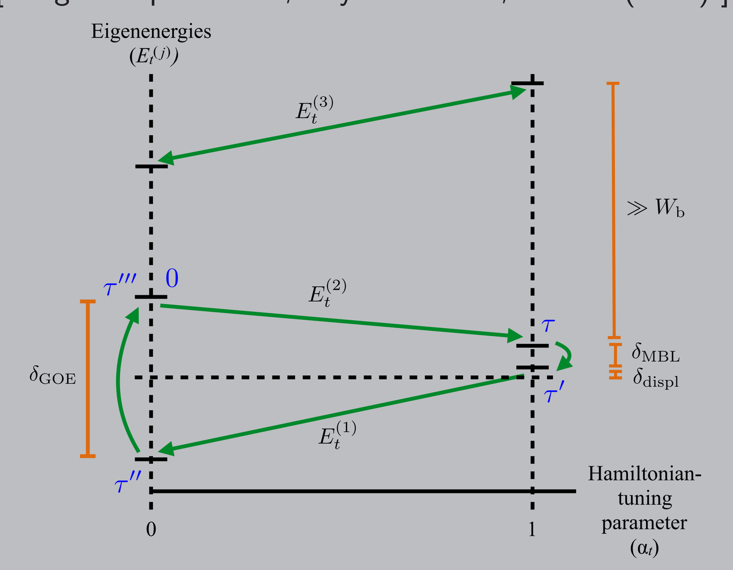
Classical Otto Engine Cycle (Internal Combustion)

[Quattroch, http://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node26.html (2006)]

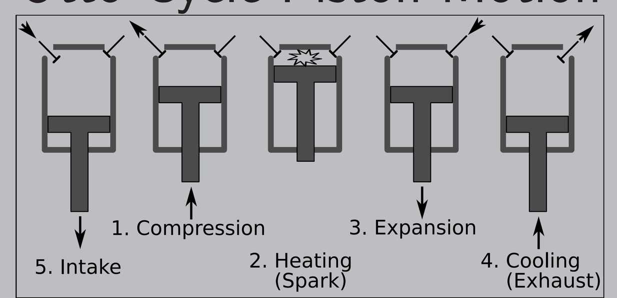


Many Body Localization (MBL) Engine Cycle

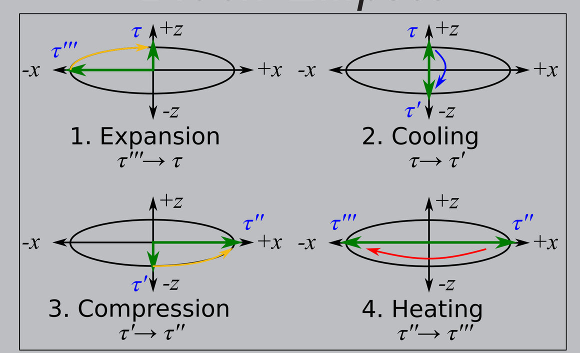
[Yunger Halpern et al., Phys.Rev.B **99**, 024203 (2019)]



Otto Cycle Piston Motion



Bloch *Ellipses*



Classical Heat and Work

$$Q = \int C_v dT$$

$$W = \int P dV$$

$$\eta = 1 - r^{1-\gamma}$$

$$r = V_1/V_2, \quad \gamma = C_v/C_p$$

Quantum Heat and Work

$$Q = \int_0^\tau \mathrm{Tr}\, \rho \frac{dH}{dt}\, dt$$

$$W = \int_0^\tau \mathrm{Tr}\, \frac{d\rho}{dt} H\, dt$$

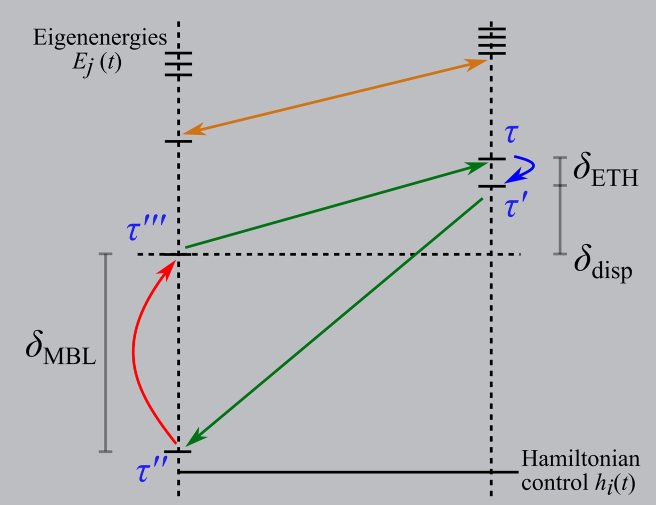
$$\eta = 1 - \delta_{\mathrm{MBL}} / \delta_{\mathrm{GOE}}$$

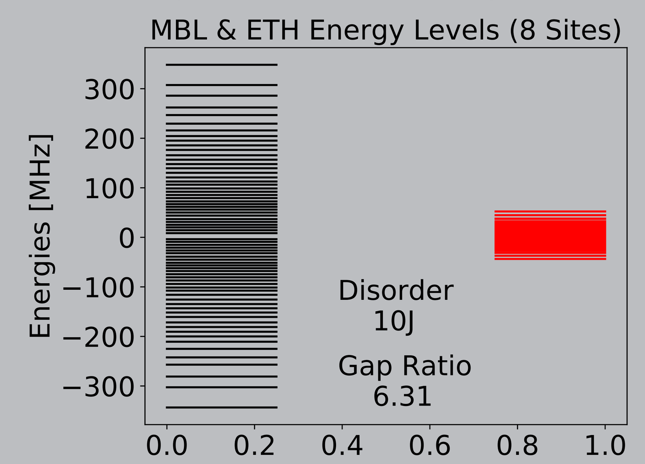
Near Ground State MBL Engine

- Near ground state configuration minimizes excursions to other states
- Heating & cooling strokes maximize and minimize energy exchange between hot & cold baths
- Same efficiency as MBL engine

$$\eta = 1 - \delta_{\mathrm{ETH}} / \delta_{\mathrm{MBL}}$$

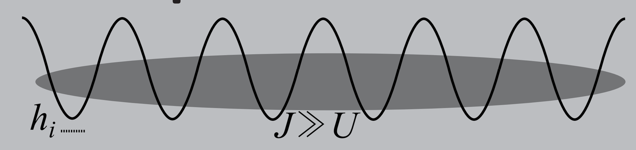
- Need to quantify adiabatic timescales for $\tau''' \to \tau$ and $\tau' \to \tau''$
- Computed spectra for MBL / ETH phases exhibits a modest gap ratio, bounding the efficiency by ~ 0.84
- Chosen maximum disorder strength has not been tested in hardware (could cause undesirable heating due to large currents on the flux lines)



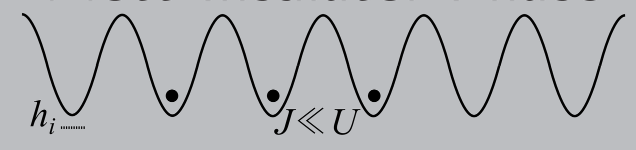


Bose Hubbard Model

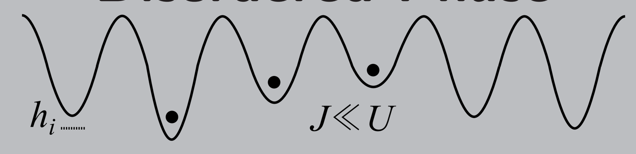
Superfluid Phase



Mott Insulator Phase



Disordered Phase



Multilevel Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \left(a_i^\dagger a_j + \mathrm{h.c.} \right) + \sum_i (h_i - \mu)\, n_i + \frac{U}{2} \sum_i n_i\, (n_i + \mathbb{1})$$

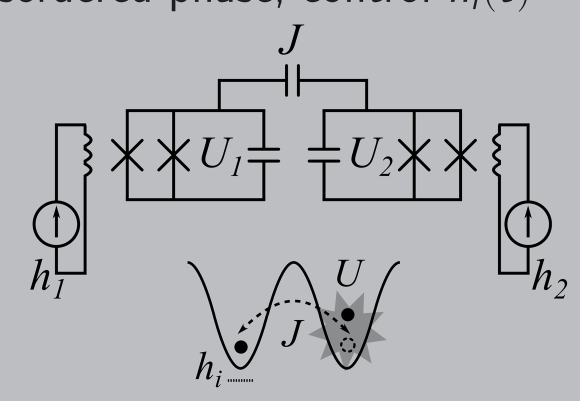
Perturbative Hamiltonian

$$H = -J \sum_{\langle ij \rangle} (\sigma_i^+ \sigma_j^- + \mathrm{h.c.}) + \sum_i (h_i - \mu)\, \sigma_i^z + \mathcal{O}\left(J / U^2 \right)$$

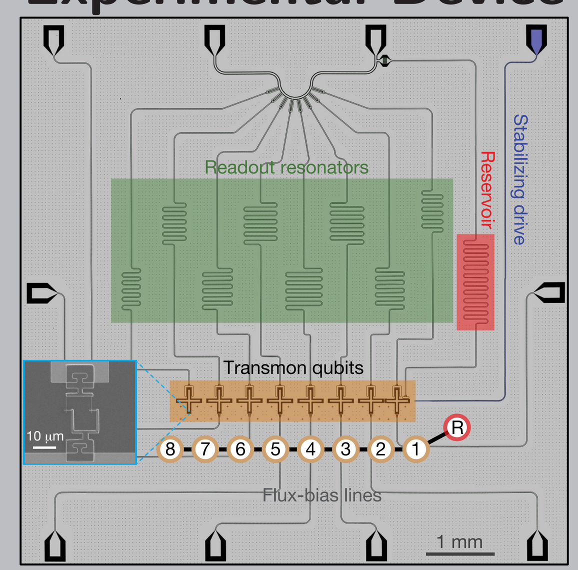
Hardware Mapping

Coupled Transmon Qubits

- Implements Bose Hubbard Model
- Defaults to Mott Insulator ($J \ll U$)
- External flux tuning can access the disordered phase, control $h_i(t)$



Experimental Device



[Ma et al., Nature **566**, 51 (2019)]

Simulation of Adiabatic Strokes

- Generate disorder realizations $\{h_i\}$

$$h_i^j \in [h_{\min},\ h_{\max}]$$

- Exact diagonalization of ETH and disordered (DIS) Hamiltonians

$$H_{\mathrm{ETH}} |\psi_n^{\mathrm{ETH}}\rangle = E_n^{\mathrm{ETH}} |\psi_n^{\mathrm{ETH}}\rangle$$

$$H_{\mathrm{DIS}} |\psi_n^{\mathrm{DIS}}\rangle = E_n^{\mathrm{DIS}} |\psi_n^{\mathrm{DIS}}\rangle$$

- Adiabatic evolution from eigenstates of $H_{\mathrm{ETH}} \leftrightarrow H_{DIS}$

$$i\partial_t |\psi(t)\rangle = H(t)\, |\psi(t)\rangle$$

$$H(t) = H_{\mathrm{ETH}} + H_{\mathrm{MBL}} C(t)$$

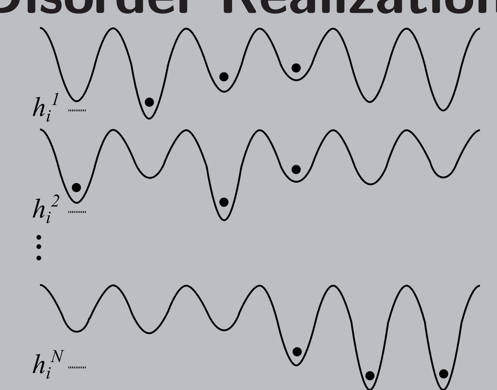
$$C(t) = \begin{cases} A(t), & \mathrm{DIS} \to \mathrm{ETH} \\ B(t), & \mathrm{ETH} \to \mathrm{DIS} \end{cases}$$

- Compute fidelity $\mathcal{F}$ and residual energy E_{res} averaged over disorders

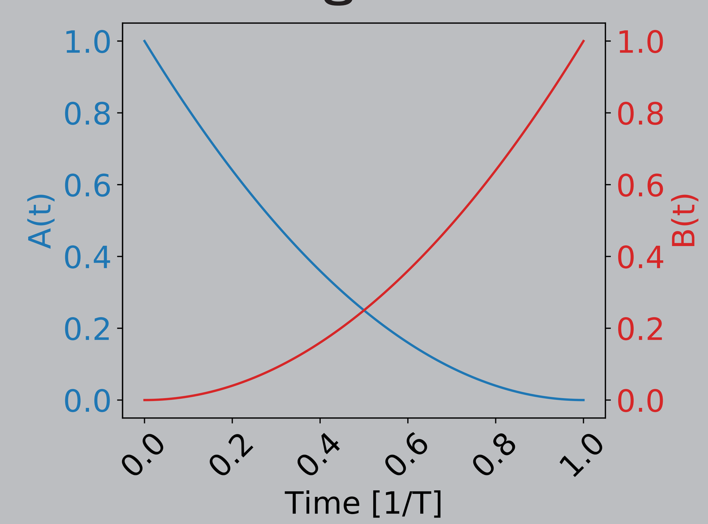
$$E_{\mathrm{res}} = |E_{\mathrm{target}} - \langle H_{\mathrm{target}} \rangle|$$

$$\mathcal{F} = |\langle \psi_{\mathrm{target}} | \psi(t) \rangle|^2$$

Disorder Realizations



Annealing Schedules

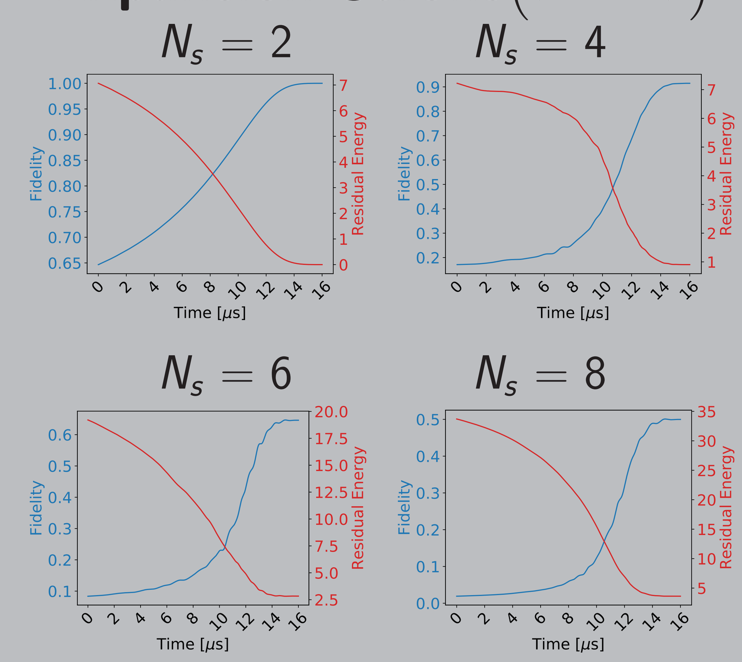


Results

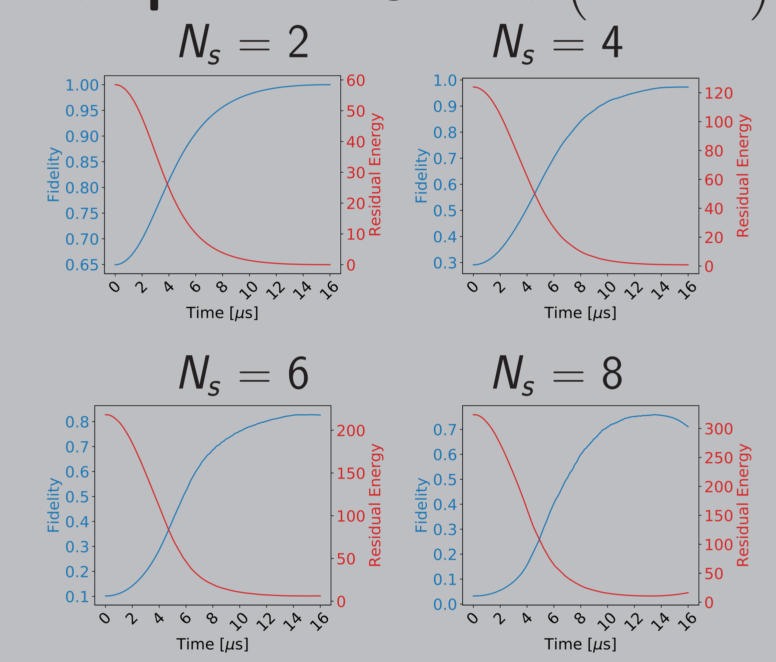
Model Parameters

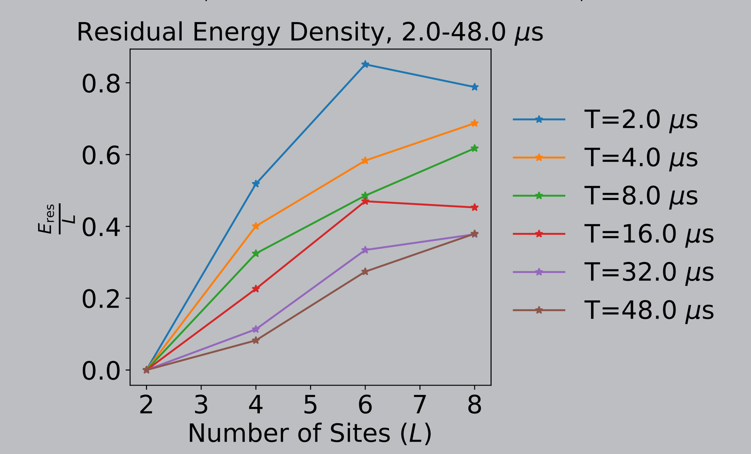
$$J/2\pi = 10\ \mathrm{MHz}, \quad U/2\pi = 250\ \mathrm{MHz}, \quad h \in [-10J,\ 10J]$$

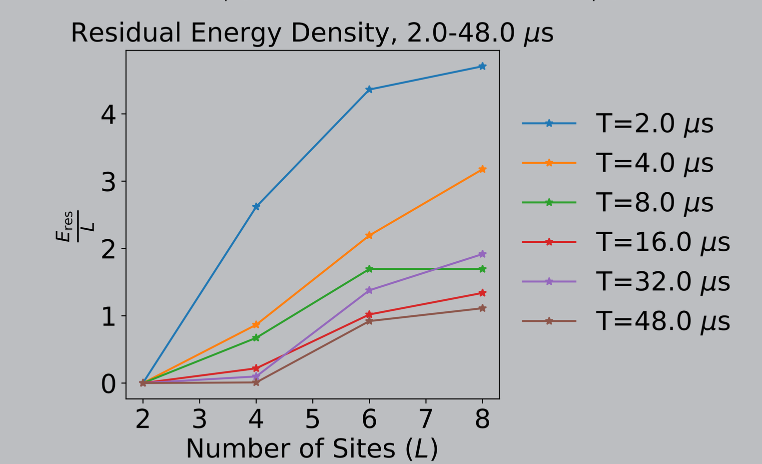
Expansion Stroke $(\tau''' \to \tau)$



Compression Stroke $(\tau' \to \tau'')$







- Results confirm that the ground state to ground state transition (compression stroke) reaches a higher fidelity in a shorter time compared to the first excited to first excited state transition (expansion stroke)
- Residual energy density offers a spatiotemporal metric for evaluating the scaling of both strokes' performance with system size and evolution time
- The required time to reach unit fidelity in the eight site case is much longer than available coherence times
- For the two and four site cases, both strokes are achievable with $\sim 40\ \mu s$ on current hardware

Future Work

- Repeat the analysis using the transverse field Ising model as the base Hamiltonian and adding disorder to J_{ij}, h_i
- Compute the heat and extract the work done by the engine using the density matrix and Hamiltonian expressions
- Discuss details of simulating the heat engine on superconducting qubits / quantum annealers with experimental collaborators

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