

Simulation of Superconducting Qubit Devices

Workshop on Microwave Cavities and Detectors
for Axion Research

Nick Materise

January 10, 2016



Outline

- Definition of a qubit
- Non-linearity in superconducting qubits and Josephson junctions
- Cavity QED and Circuit QED
- Black box Circuit Quantization
- Types of Superconducting Qubits
- Physical realization of superconducting circuits
- Simulating RF components of qubits in COMSOL

Qubits

- A quantum “bit” or two level system / *effective* two level system with addressable energy levels
- In some cases, a qubit can be treated as a harmonic oscillator with non-linearly spaced levels
- Level spacing due to anharmonicity from non-linearity(ies), allows for designs that minimize leakage to higher excited states of the qubit(s)

Source of Non-linearity: Josephson Junction

- DC Josephson Effect – B. Josephson, 1962¹
 - Non-zero periodic current, due to tunneling Cooper Pairs across an SIS (superconductor-insulator-superconductor) junction
 - The current varies periodically in the phase difference across the junction, acting as a macroscopic quantum variable
 - Josephson Current and Voltage Equations

¹ B.D. Josephson, Phys.Lett. **1**, 7 (1962)

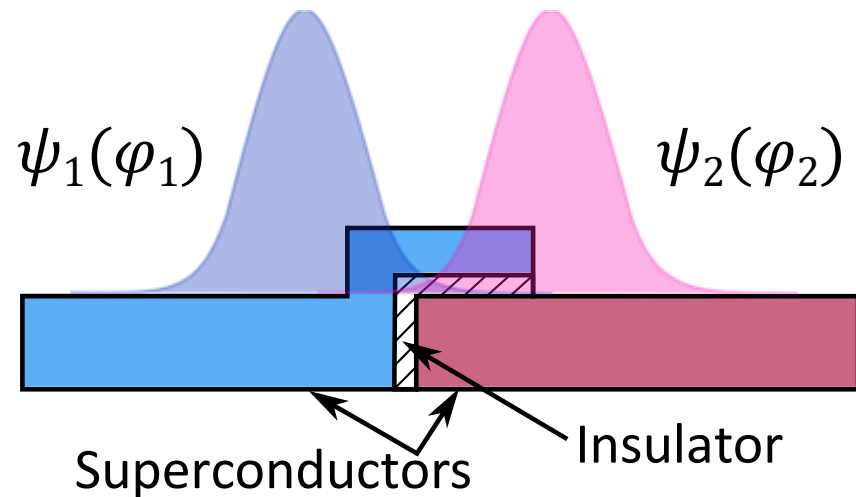
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$$I = I_c \sin \varphi$$

$$V = \frac{\hbar}{2e} \frac{d\varphi}{dt} = \frac{d\phi}{dt}$$

$$\varphi = \varphi_1 - \varphi_2$$



¹B.D. Josephson, Phys.Lett. **1**, 7 (1962)

IV Characteristics of Josephson Junctions

- The DC current in an SIS junction is given at zero temperature²

$$I_{\text{dc}} = \text{Im} \{j_2(\omega)\}, \quad I_{\text{dc}} \sim \text{sgn}(\omega) K_0 \left(\left(\frac{x^2 - 1}{x^2 - \delta^2} \right)^{1/2} \right), \quad x = \frac{|\omega|}{\Delta_1 + \Delta_2}, \quad \delta = \frac{\Delta_1 - \Delta_2}{\Delta_1 + \Delta_2}$$

- where K_0 is the zero-th order modified Bessel function of the first kind, Δ_1, Δ_2 are the superconducting gap energies of the superconducting leads

² N.R. Werthamer, Phys. Rev. **147**, 255 (1966)

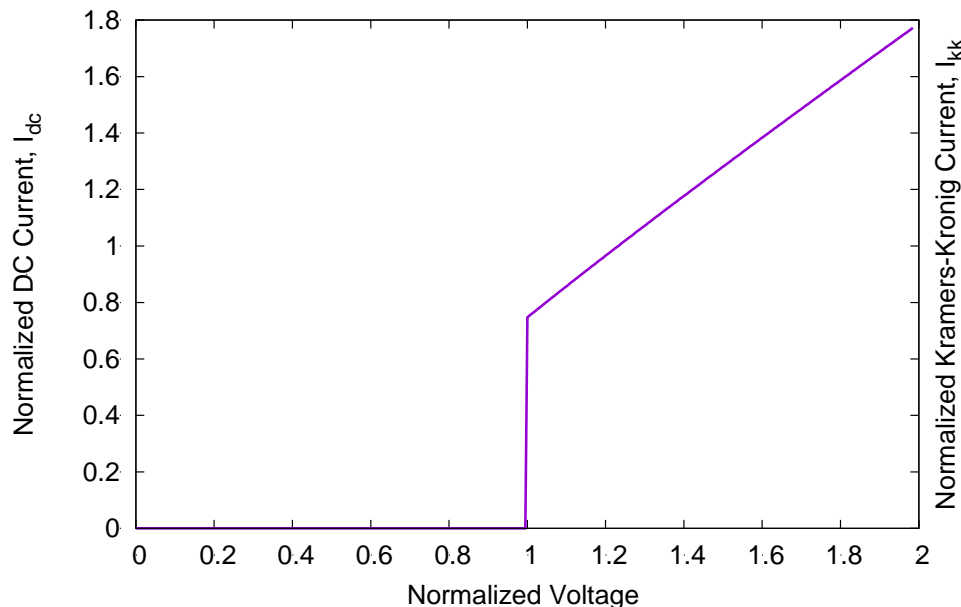
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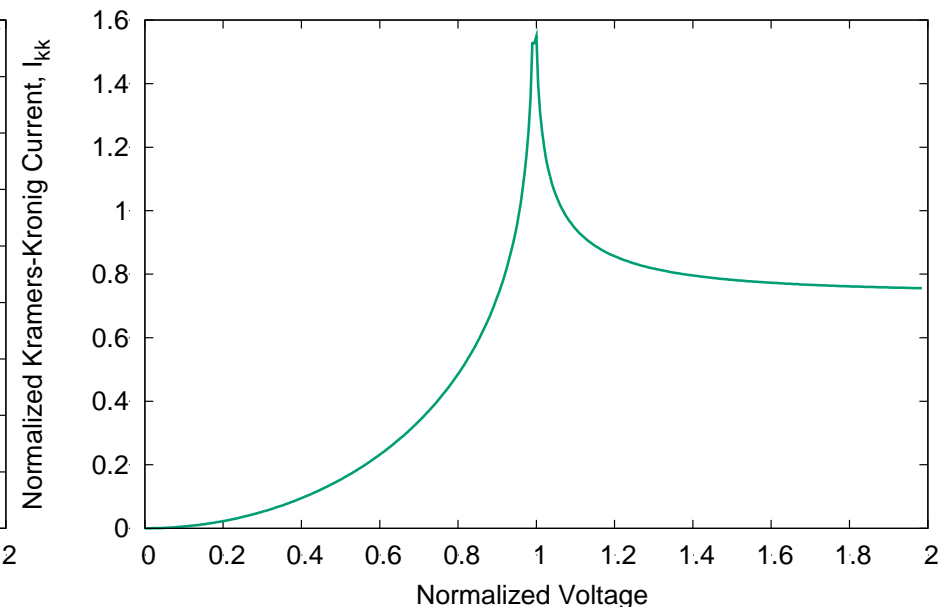
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Normalized IV Curve for Al-Al₂O₃-Al Josephson Junction.



Normalized Kramers-Kronig Curve for Al-Al₂O₃-Al Josephson Junction.



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Josephson Junction Circuit Model

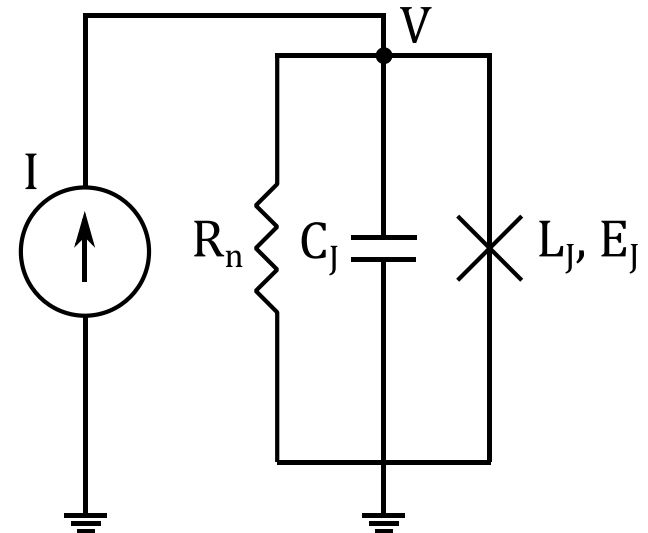
- Josephson Junctions can be approximated by linear, passive circuit elements shunting a non-linear inductance L_J
 - RCSJ Model (Resistance and Capacitive Shunted Junction)³
 - Useful model for including simple non-linear behavior in classical simulations, e.g. COMSOL
 - From Kirchhoff's current law, the current flowing through each element in the circuit is given by

³ D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.`

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$$I = \frac{V}{R_n} + C_J \frac{dV}{dt} + I_c \sin(\varphi)$$



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Circuit Quantum Electrodynamics (cQED)

- Use Josephson Junctions as a source of non-linearity to realize macroscopic quantum systems
- Borrow concepts from the optics community, e.g. cavity QED to implement familiar systems
- Atom in a resonant cavity is the most basic model

Cavity QED and Model Hamiltonians

- Cavity QED: two level atomic system trapped in a mirrored, high finesse resonant cavity
- Follows the Jaynes-Cummings Hamiltonian³

$$\hat{H} = \underbrace{\hbar\omega_r (\hat{a}^\dagger \hat{a} + 1/2)}_{\text{EM field quantization}} + \underbrace{\hbar\omega_a \hat{\sigma}_z / 2}_{\text{spin-1/2 atom}} + \underbrace{\hbar g (\hat{a}^\dagger \hat{\sigma}_- + \hat{a} \hat{\sigma}_+)}_{\text{atom-cavity interaction}}$$

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$2g =$ Vacuum Rabi Frequency

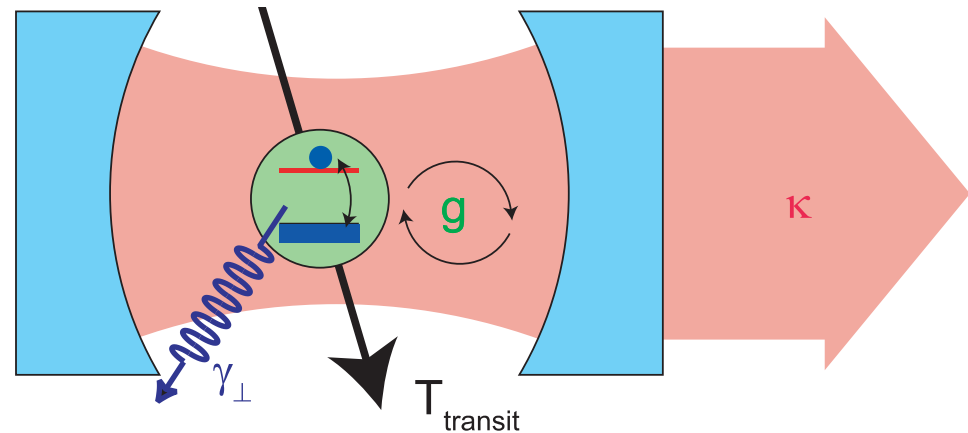
$\kappa =$ Cavity Decay Rate

$\gamma_\perp =$ Transverse Decay Rate

$T_{\text{transit}} =$ Time for atom to leave cavity

³ D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

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Atom trapped in a cavity with photon emission, atom-cavity dipole coupling, and atom transit time shown⁴.

Cavity QED and Circuit QED, from optics to RF

Cavity QED	Circuit QED
Two Level Atom	Artificial atom, truncated to two levels
High Finesse Cavity	High Q Cavity / Planar Resonator
Small transition dipole moment	Arbitrarily large transition dipole moment, e.g. strong coupling regime
$1/\kappa, 1/\gamma$	T_1, T_2

- Large dipole moment couples the qubit well to the cavity in superconducting qubits: coupling strength and energy levels are *tunable* by design or in situ

Cavity QED and Circuit QED, Device Comparison

Parameter	Symbol	Cavity QED ³	Circuit QED ^{3, 5}
Resonator, Qubit Frequencies	$\omega_r, \omega_q / 2\pi$	~ 50 GHz	~ 5 GHz
Transition Dipole Moment	d / ea_0	~ 1	~ 10 ⁴
Relaxation Time	T_1	30ms	60 μ s
Decoherence Time	T_2	~1 ms	~10-20 μ s

- Large dipole moment couples the qubit well to the cavity in superconducting qubits: coupling strength and energy levels are *tunable*
- Trapped atoms in cavities have *longer coherence times, not tunable*, weakly coupled to the cavity for measurement

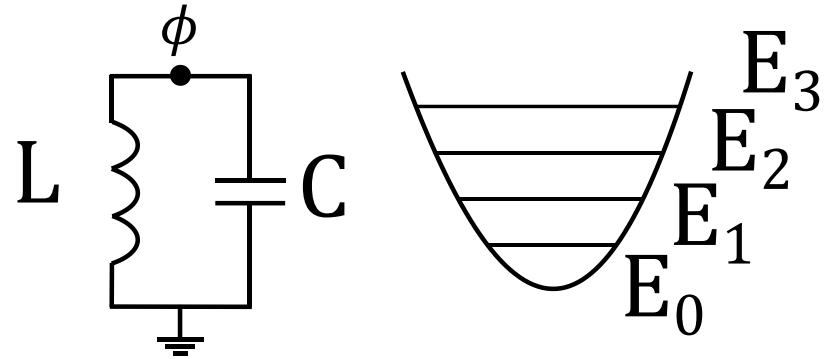
³ D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

⁵ H. Paik, et al., Phys. Rev. Lett. **107**, 240501 (2011)

Quantizing Simple Circuits

- Simplest model is an LC-resonator treated as a quantum harmonic oscillator with classical Lagrangian, Hamiltonian, and quantized operators³

$$\mathcal{L}(\phi, \dot{\phi}) = \frac{1}{2}C\dot{\phi}^2 - \frac{\phi^2}{2L}$$



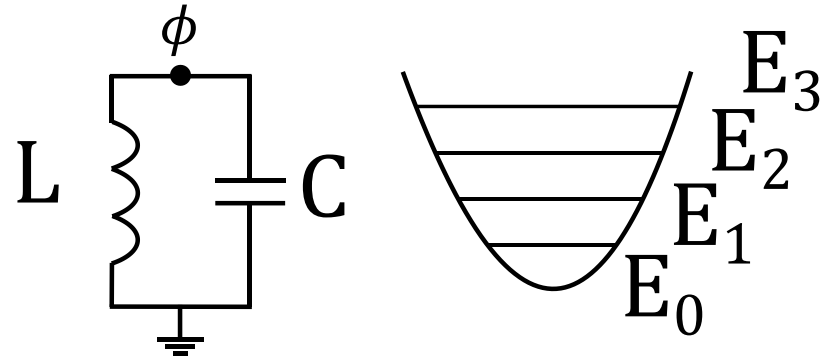
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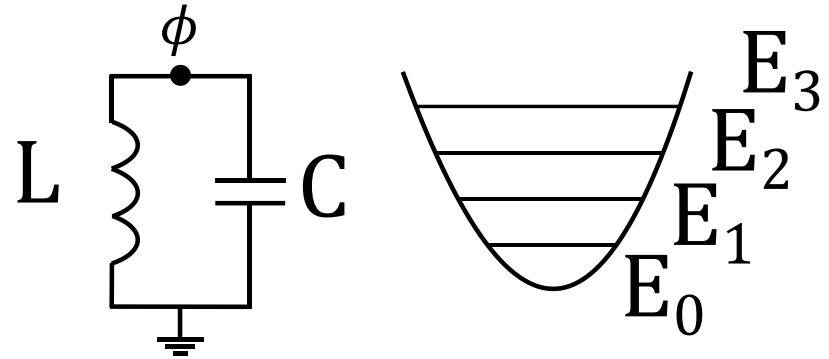
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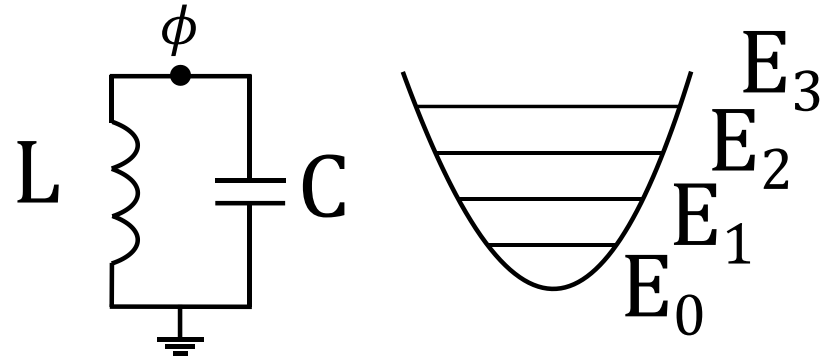
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$$\omega^2 = 1/(LC)$$



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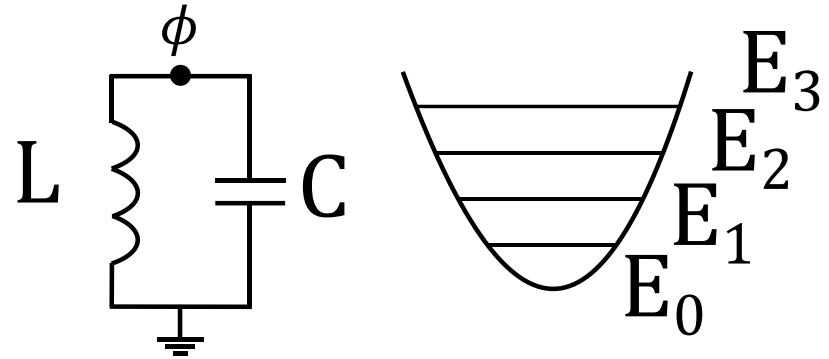
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$$\omega^2 = 1/(LC)$$

$$\implies \hat{H} = \boxed{\hbar\omega (\hat{a}^\dagger \hat{a} + 1/2)}$$



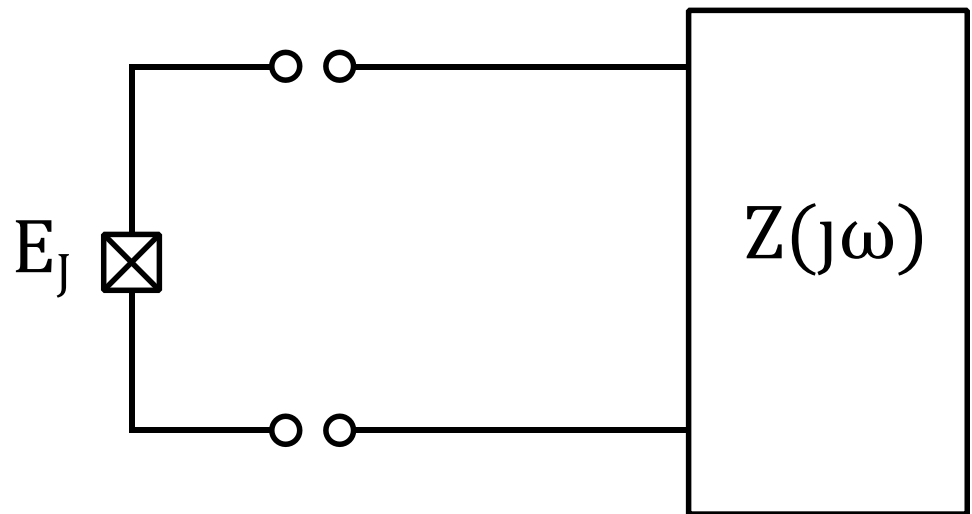
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Black box Circuit Quantization

- Idea is to extract all linear components of the qubit and microwave circuitry by synthesizing an equivalent passive electrical network
- The network is obtained by computing the S-parameters of a device using FEM software (COMSOL, HFSS) and converting them to an impedance, $Z(j\omega)$

$$Z = (\mathbb{1} + S) (\mathbb{1} - S)^{-1}$$

$$\mathbb{1} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}$$



Black box Circuit Quantization—Vector Fit

- The impedance function is fit to a pole-residue expansion following the Vector Fit procedure, a least squares fit to a rational function of the form⁶

$$Z(s) \simeq \sum_{k=1}^M \frac{R_k}{s - s_k} + d + es$$

- From this form, there are two synthesis approaches with two quantization schemes
 - Lossy Foster approach (approximate circuit synthesis)⁷
 - Brune exact synthesis approach⁸

⁶ B. Gustavsen et al., IEEE Tran on Power Delivery, 14(3):1052–1061, Jul 1999

⁷ F. Solgun et al. Phys.Rev.B **90**, 134504 (2014)

⁸ S. E. Nigg et al. Phys.Rev.Lett. **108**, 240502 (2012)

Black box Circuit Quantization—Lossy Foster

- Taking the constant term $d = 0$ and excluding the pole at $s = 1$ or setting $e = 0$ leaves the rational function with poles and residues R_k, s_k ⁷

$$Z(s) = \sum_{k=1}^M \frac{R_k}{s - s_k}, \quad s_k = \xi_k + j\omega_k, \quad R_k = a_k + jb_k$$

- Expanding the k -th component of $Z(s)$ in partial fractions and taking the low loss limit, $a_k, b_k \ll 1$

$$Z_k(s) = \frac{R_k}{s - s_k} + \frac{R_k^*}{s - s_k^*} \simeq \frac{2a_k s}{s^2 - 2\xi_k s + \omega_k^2}$$

$$\Rightarrow Z_k(s) = \frac{\frac{\omega_k R_k}{Q_k} s}{s^2 + \frac{\omega_k}{Q_k} s + \omega_k^2}$$

RLC Tank Circuit!

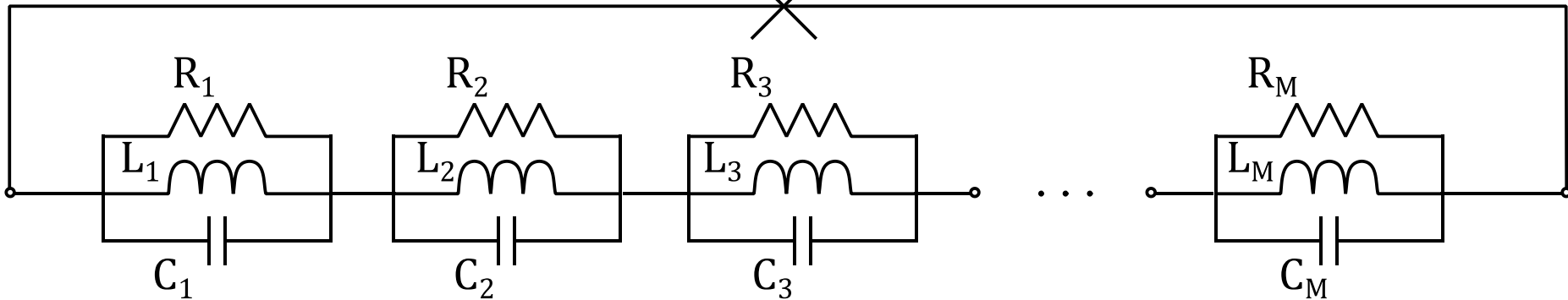
⁷ F. Solgun et al. Phys.Rev.B **90**, 134504 (2014)

Black box Circuit Quantization—Lossy Foster

- Main result of the Lossy Foster treatment is a set of uncoupled harmonic oscillators as a series of RLC circuits
- Circuit elements in terms of the real and imaginary components of the poles and residues⁷

$$\omega_k^2 = \frac{1}{L_k C_k}, \quad Q_k = \omega_k R_k C_k = -\omega_k / 2\xi_k, \quad R_k = -a_k / \xi_k$$

$$\varphi = \sum_m \phi_m$$



⁷ F. Solgun et al. Phys.Rev.B **90**, 134504 (2014)

⁹ J. Bourassa et al. Phys.Rev.A **86**, 013814 (2012)

Black box Circuit Quantization—Lossy Foster

- From the circuit elements, a lossless Hamiltonian is obtained by taking the limit $R_k \rightarrow 1 \delta_k$
- The LC circuits are quantized as harmonic oscillators giving the linear Hamiltonian⁸

$$\hat{H}_0 = \sum_{k=1}^M \hbar \omega_k \left(\hat{a}_k^\dagger \hat{a}_k + 1/2 \right)$$

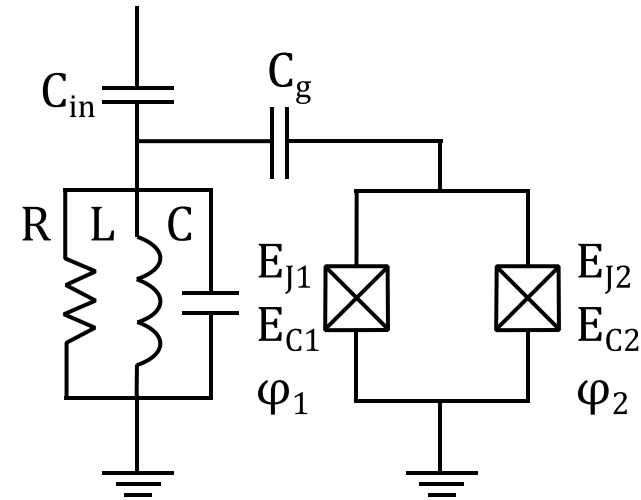
- A non-linear Hamiltonian accounts for the qubit and its coupling to the harmonic modes

$$\hat{H}_{\text{nl}} = E_J \left(1 - \cos \hat{\varphi} - \frac{\hat{\varphi}^2}{2} \right), \quad \hat{\varphi} = \sum_{k=1}^M \sqrt{\frac{\hbar}{2\omega_k C_k}} \left(\hat{a}_k + \hat{a}_k^\dagger \right)$$

⁸ S.E. Nigg et al. Phys.Rev.Lett. **108**, 240502 (2012)

Types of Superconducting Qubits: Charge Qubits

- Early Charge qubit – Cooper Pair Box (CPB)
 - Cooper pairs tunnel across the junction leading to a charge number operator
 - Typical implementations include a resonator that plays the role of a cavity
 - Hamiltonian in the charge basis for single Josephson junction³



$$\hat{H}_{\text{CPB}} = 4E_C \left(\hat{N} - n_g/2 \right)^2 + \frac{1}{2} E_J \sum_n (|n\rangle \langle n+1| + |n+1\rangle \langle n|)$$

- “Split” CPB including RLC resonator and coupling³

$$\hat{H}_{\text{CPB+RLC}} = \underbrace{4E_C \left(\hat{N} - n_g/2 \right)^2}_{\text{CPB term}} + \underbrace{\frac{4E_C C_g \hat{V} (2V_g + \hat{V})}{e}}_{\text{junction capacitance}} - \underbrace{\frac{4C C_g \hat{V} \hat{N}}{e}}_{\text{RLC-CPB coupling}}$$

³ D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

Types of Superconducting Qubits: Charge Qubits

- Early Charge qubit – Cooper Pair Box (CPB)

- Rotating wave approximation (RWA) and Jaynes-Cummings Hamiltonian

- Approximate number and charge number operators as Pauli operators³

$$\hat{N} \approx \hat{\sigma}_z/2, \quad (|n\rangle\langle n+1| + |n+1\rangle\langle n|) \approx \hat{\sigma}_x$$

- Expand voltage operator, V , apply RWA to coupling term and substitute the qubit plasma frequency, $\omega_p = (L_J C_J)^{-1/2}$

$$\hat{V} = \sqrt{\frac{\hbar\omega_r}{2C}} (\hat{a} + \hat{a}^\dagger), \quad -\frac{4E_C C_g \hat{V} \hat{N}}{e} = 2\hbar g (\hat{a} + \hat{a}^\dagger) \hat{\sigma}_x$$

$$2\hbar g (\hat{a} + \hat{a}^\dagger) \hat{\sigma}_x \approx \hbar g (\hat{a} \hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-)$$

$$\boxed{\hat{H}_{CPB+RLC} \approx \hbar\omega_r \hat{a}^\dagger \hat{a} + \hbar\omega_p \hat{\sigma}_z/2 + \hbar g (\hat{a} \hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-)}$$

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**Reclaims Jaynes-Cummings Hamiltonian

³ D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

Types of Superconducting Qubits: Transmon

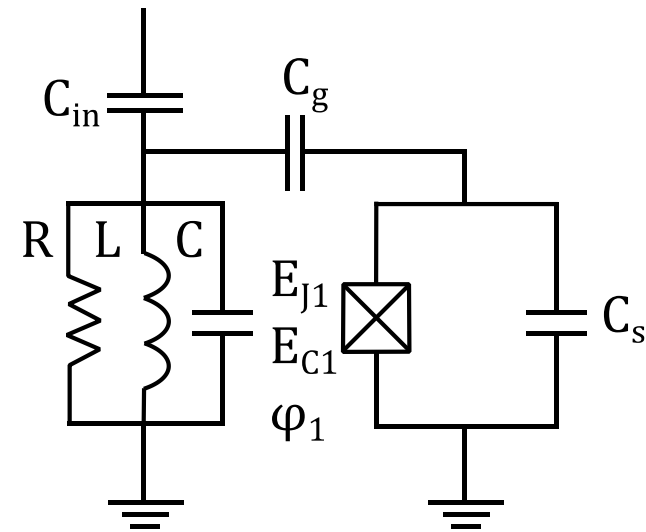
- Transmon, an improved charge qubit
 - Shunt capacitor reduces sensitivity to charge noise
 - Flatter energy levels, weakly anharmonic
 - Hamiltonian, qubit + resonator³

$$\hat{H}_{\text{trans}} = -4E_C \frac{\partial^2}{\partial \varphi^2} - E_J \cos(\varphi)$$

$$\approx -4E_C \frac{\partial^2}{\partial \varphi^2} - E_J \left(1 - \frac{\varphi^2}{2} + \frac{\varphi^4}{4!} + \mathcal{O}(\varphi^6) \right)$$

$$\varphi = \frac{1}{\sqrt{2}} \left(\frac{8E_C}{E_J} \right)^{1/4} (\hat{b} + \hat{b}^\dagger)$$

$$\hat{N} = \frac{1}{\sqrt{2}} \left(\frac{E_J}{8E_C} \right)^{1/4} (\hat{b} - \hat{b}^\dagger)$$



³ D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

Types of Superconducting Qubits: Transmon

- Transmon, an improved charge qubit
 - Hamiltonian in the energy basis, anharmonic oscillator³

$$\hat{H}_{\text{trans}} \approx \hbar\omega_p \left(\hat{b}^\dagger \hat{b} + 1/2 \right) - \frac{E_C}{12} \left(\hat{b} + \hat{b}^\dagger \right)^4$$

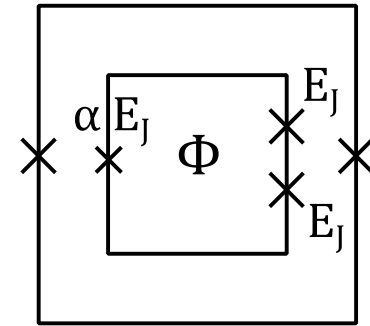
- Including resonator and coupling term follow a similar treatment as the CPB qubit
- Kerr and cross Kerr terms may be included when expanding the fourth power in b

³ D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

Types of Superconducting Qubits: Flux Qubits

- Flux qubit

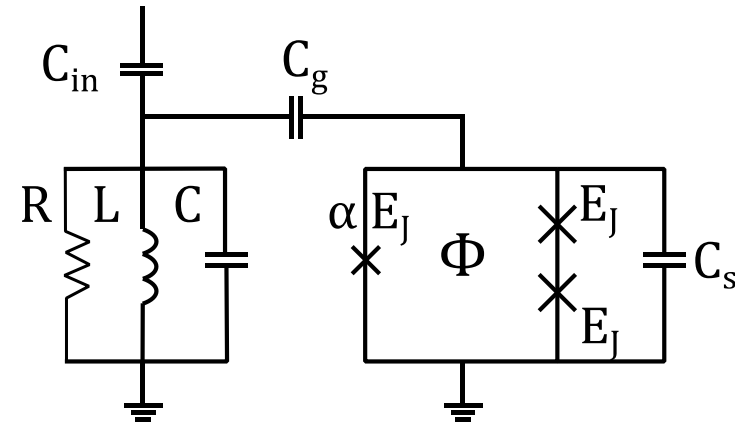
- Flux threading a loop is quantized; DC SQUID biases the qubit
- Persistent current I_p forms in the superconducting loop
- Hamiltonian, with fixed gap $\Delta^{10, 11}$



$$\hat{H} = I_p \Phi_0 \left(\Phi / \Phi_0 - (n + 1/2) \right) \hat{\sigma}_z + \hbar \Delta \hat{\sigma}_x$$

- Capacitively shunted flux qubit

- Shunt the flux qubit with a large capacitor, similar to the transmon for charge qubits
- Hamiltonian, with resonator¹²



$$\hat{H} \approx \hbar \omega_q (\Phi_b) \hat{\sigma}_z / 2 + \hbar \omega_r (\hat{a}^\dagger \hat{a} + 1/2) + \hbar \chi (\Phi_b) (\hat{a}^\dagger \hat{a} + 1/2) \hat{\sigma}_z$$

¹⁰ M.J. Schwarz et al. *New Journal of Physics* **15** (2013) 045001

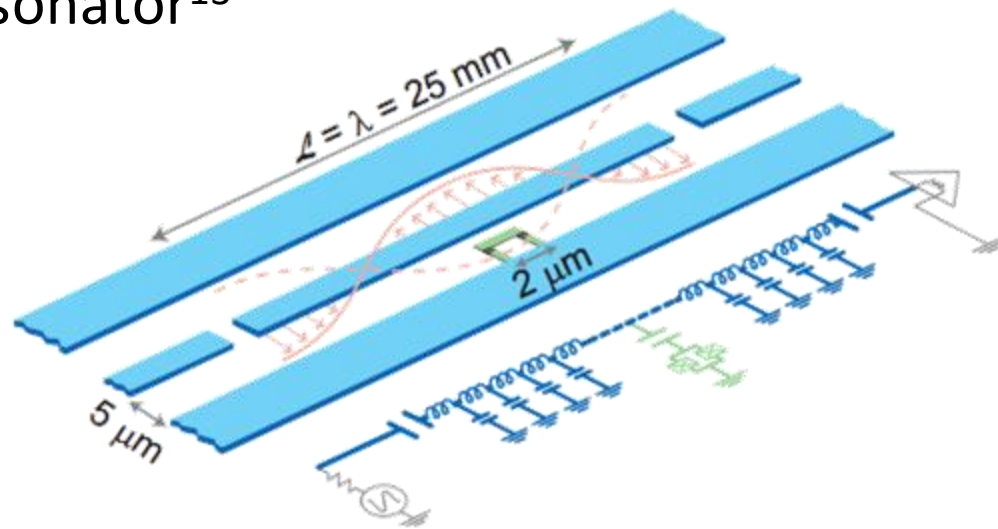
¹¹ T.P. Orlando, *Phys.Rev.B* **60**, 15398 (1999)

¹² F. Yei et al, *Nature Communications* **7** (2016)

Physical Designs: CPW + CPB = Cavity + Atom

- Implemented as Josephson Junction capacitively coupled to transmission line resonator (coplanar waveguide, CPW)
 - Transmission Line Resonator ~ Cavity
 - 2D Planar or 3D cavity couples qubit to drive and readout
- Dipole moment, d , in terms of the magnitude of the applied voltage V_0 , CPW conductor width w , electric field magnitude E_0 , and coupling of the qubit to the resonator¹³

$$\hbar g = (ew) \frac{1}{w} V_0 = dE_0$$

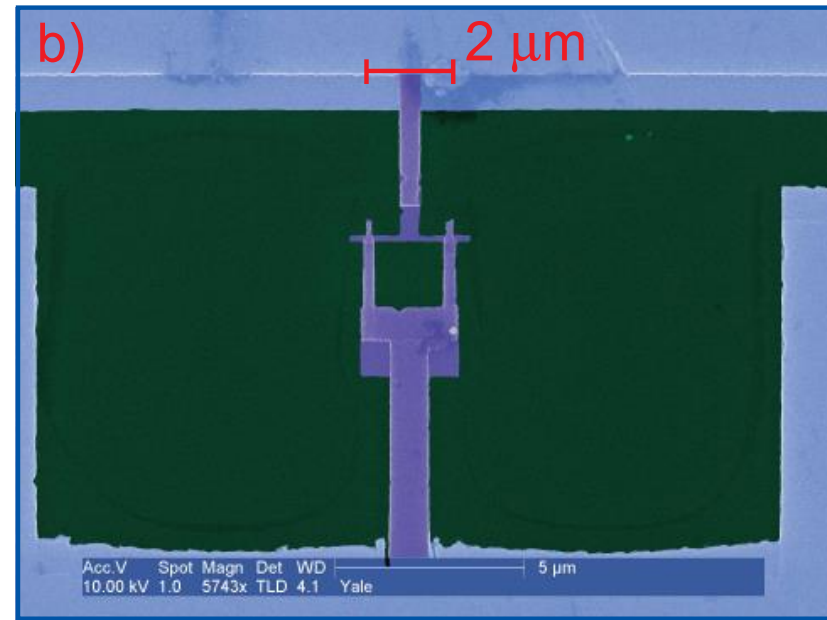
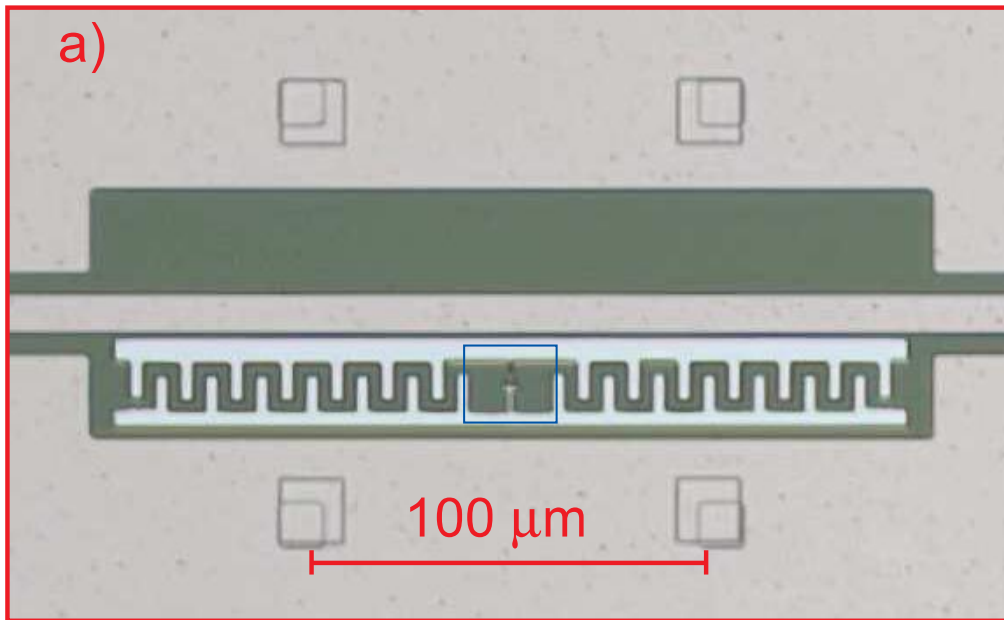


¹³ A. Blais et al., Phys. Rev. A **69**, 062320 (2004)

Coplanar waveguide resonator and lumped circuit¹³

Physical Designs: Transmon

- Similar in design to CPB with the following modifications
 - Shunt capacitance implemented with an interdigitated capacitor or sufficiently large gap of exposed substrate between conductors

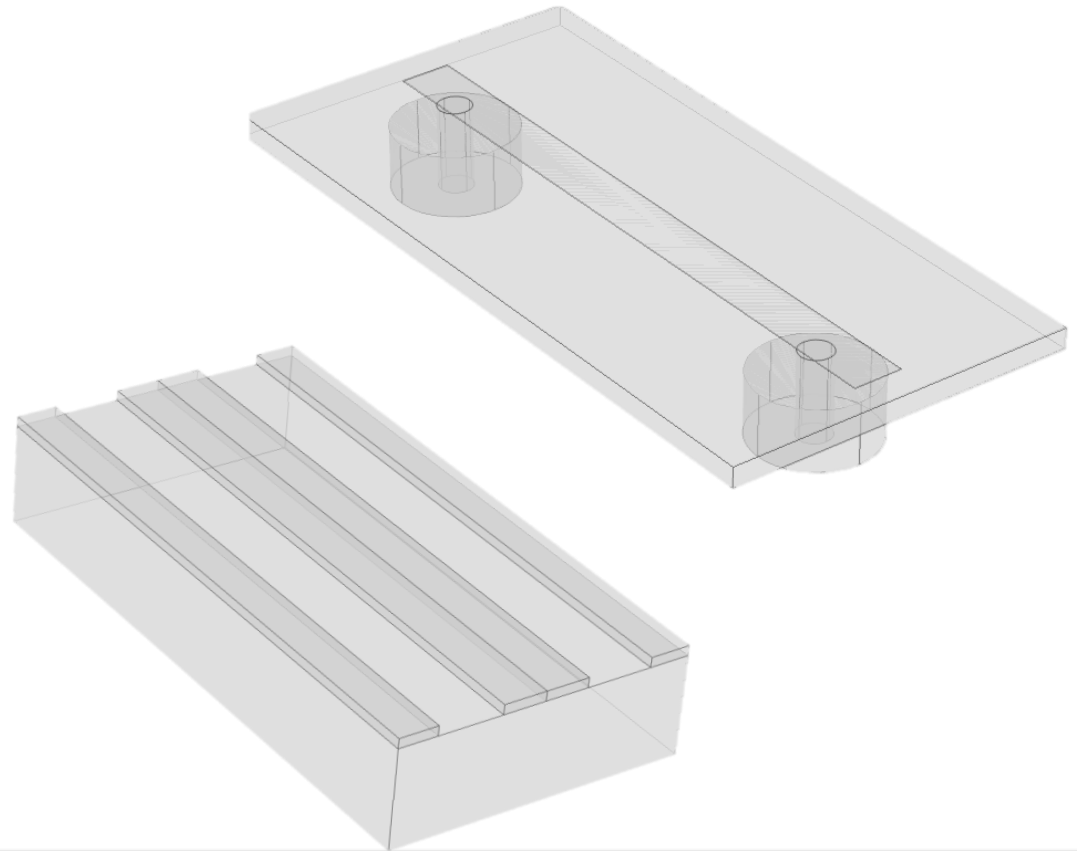


Micrograph of resonator and transmon reproduced from³

³ D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

COMSOL RF Simulation Building Blocks

- Model systems used to develop more accurate descriptions of the microwave circuits that constitute a qubit
- Model Progression
 1. Microstrip transmission line
 2. Coplanar Waveguide (CPW)

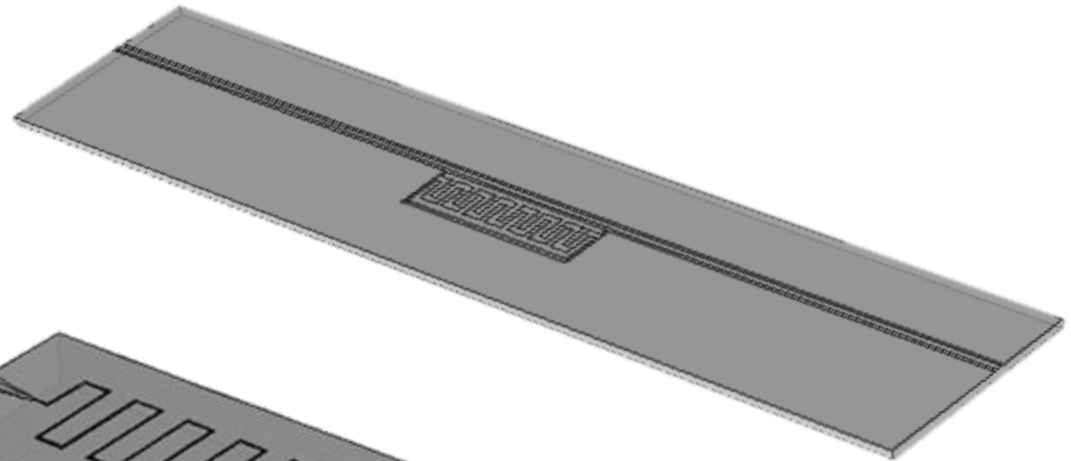


COMSOL RF Simulation Building Blocks

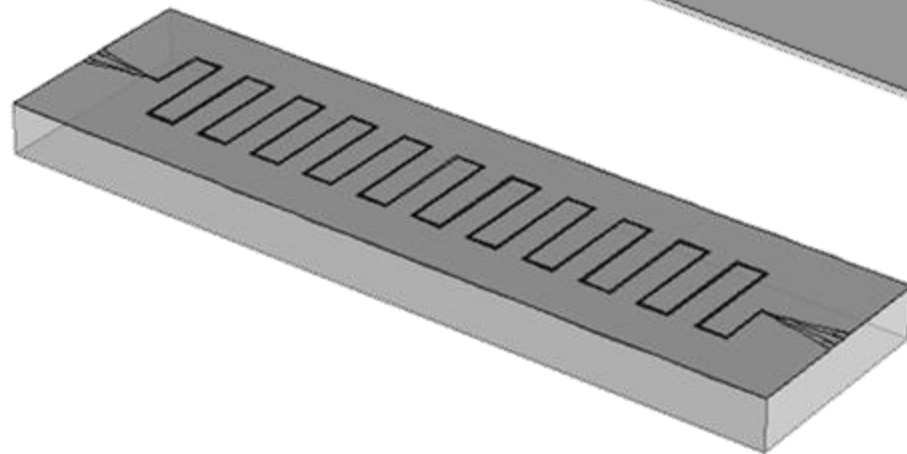
- Model systems used to develop more accurate descriptions of the microwave circuits that constitute a qubit

- Model Progression

1. Interdigitated Capacitor (IDC)



2. Meanderline resonator

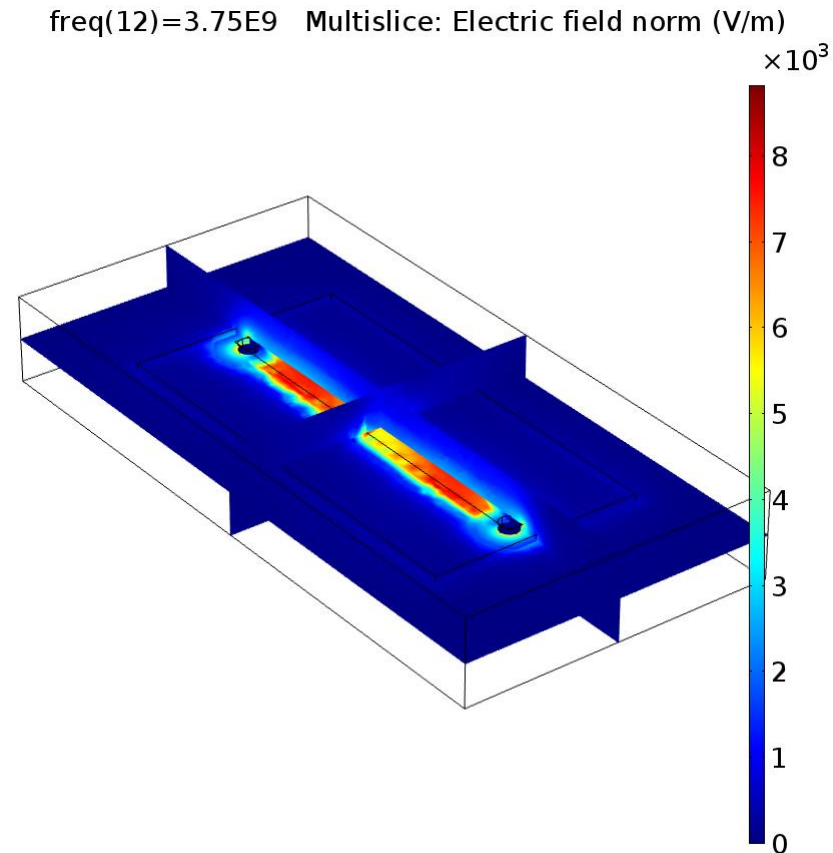


Microstripline Resonator

- Electric field norm and characteristic impedance, Z_0
- Characteristic impedance is given by

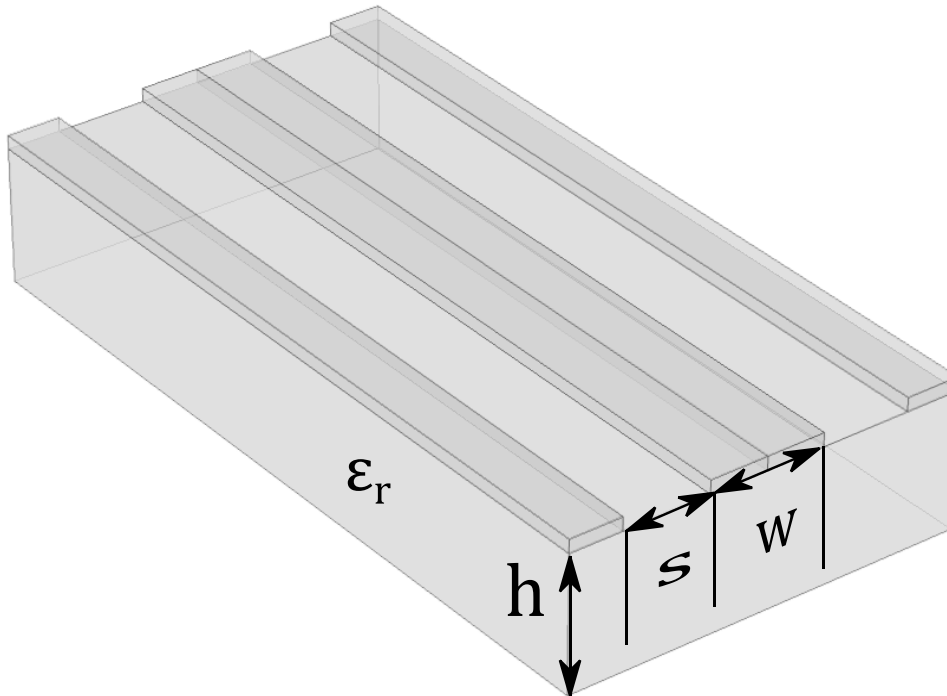
$$Z_0 = \frac{87}{\sqrt{\epsilon_r + 1.41}} \ln \left(\frac{5.98h}{0.8w + t} \right)$$

where h is the substrate thickness, t is the thickness of the microstrip, w is the width of the strip



Coplanar Waveguide

- Coplanar waveguide used as a resonant coupling structure, i.e. *cavity* with the qubit



- Characteristic Impedance from conformal mapping¹⁴:

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_{\text{eff}}}} \frac{K(k'_0)}{K(k_0)}$$

$$\epsilon_{\text{eff}} = 1 + \frac{\epsilon_r - 1}{2} \frac{K(k_1)}{K(k'_1)} \frac{K(k'_0)}{K(k_0)}$$

$$k_0 = \frac{s}{s + 2w}, \quad k'_0 = \sqrt{1 - k_0^2}$$

$$k_1 = \frac{\sinh(\pi s / 4h)}{\sinh(\pi (s + 2w) / 4h)}$$

$$k'_1 = \sqrt{1 - k_1^2}$$

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}}$$

¹⁴ Raine N Simons. Coplanar Waveguide Circuits, Components, and Systems, chapter 2. Wiley Series in Microwave and Optical Engineering. Wiley, 2001.

COMSOL RF Module Demo: Work planes

The screenshot displays the COMSOL Multiphysics interface for a meander line model. The title bar indicates the file name is `cpw_meander.mph`. The software is set to the `Electromagnetic Waves, Frequency Domain (emw)` module, with a `Mesh 1` and `Study 1` configuration. The `Electric Field (emw)` physics interface is active.

The **Model Builder** tree on the left shows the hierarchy of the model. Under `Geometry 1`, the `Metal Layer Work Plane (wp1)` is expanded to show `Plane Geometry`. The `Left Meander Center Conductor y1.1 (r7)` is selected, and its **Settings** are shown in the middle panel. The settings include:

- Label:** Left Meander Center Conductor y1.1
- Object Type:** Solid
- Size and Shape:** Width: `w_cpw` m, Height: `ly_turns` m
- Position:** Base: Center, $xw: -sub_x/2 + l_edge_taper + s_fing + 2 * l_fing + lx_turns + lx_turns/2 + 3 * w_cpw/2$ m, $yw: -w_cpw/2 + w_cpw$ m
- Rotation Angle:** Rotation: 0 deg
- Layers:** Contribute to: None
- Selections of Resulting Entities:** Show in 3D: Domain selection

The **Graphics** window on the right shows a 2D plot of the meander line geometry. The x-axis ranges from -0.004 to 0.003, and the y-axis ranges from -4.5 to 4.5, with a scale factor of $\times 10^{-3}$. The plot shows a meander line with a central conductor and ground planes.

The **Messages** window at the bottom shows the following text:

```
COMSOL Multiphysics 5.2.1.152
Finalized geometry is empty.
Opened file: cpw_meander.mph
```

1.21 GB | 6.06 GB

COMSOL RF Module Demo: Work planes

The screenshot displays the COMSOL Multiphysics interface for a model named 'cpw_meander.mph'. The Model Builder pane on the left is highlighted with a red box and contains a tree view of the model's structure, including Global Definitions, Materials, Component 1 (comp1), and Geometry 1. The Properties pane in the center shows the settings for the selected 'Left Meander Center Conductor y1.1', including its Object Type (Solid), Size and Shape (Width: w_cpw, Height: ly_turns), Position (Base: Center, X: $-sub_x/2 + l_edge_taper + s_fing + 2*l_fing + lx_turns + lx_turns/2 + 3*w_cpw/2$, Y: $-w_cpw/2 + w_cpw$), and Rotation Angle (0 deg). The Graphics window on the right shows a 2D plot of the meander line geometry on a grid, with axes ranging from -0.004 to 0.003 on the x-axis and -4.5 to 4.5 on the y-axis. The right sidebar shows the 'Recently Used' list and 'Physics interfaces in study' section.

Model builder pane includes the geometry, materials, and physics

1.21 GB | 6.06 GB

COMSOL Multiphysics 5.2.1.152
Finalized geometry is empty.
Opened file: cpw_meander.mph

COMSOL RF Module Demo: Arrays

The screenshot displays the COMSOL Multiphysics interface for configuring an Array component. The left pane shows the 'Settings' window for the 'Array' component, with the label 'Meander Line Center Conductor Middle Array'. The 'Input' section lists objects r6, r7, r8, and r9. The 'Size' section is set to 'Rectangular' with 'xw size' as 'n_turns-1' and 'yw size' as '1'. The 'Displacement' section has 'xw' set to $2 * i_x_turns + 2 * w_cpw$ and 'yw' set to '0'. The 'Selections of Resulting Entities' section is set to 'None'. The right pane shows a 2D plot of the array geometry, with axes ranging from -3.8 to -2.0 on the x-axis and -1.0 to 1.2 on the y-axis, both scaled by 10^{-3} . The plot shows a meander line structure with a central conductor and a meander line. The rightmost part of the structure is highlighted with a blue border. The bottom pane shows the 'Messages' window with the following text: 'COMSOL Multiphysics 5.2.1.152', 'Finalized geometry is empty.', and 'Opened file: cpw_meander.mph'. The bottom status bar indicates '1.19 GB | 6.05 GB'.

COMSOL RF Module Demo: Arrays

The screenshot displays the COMSOL Multiphysics interface for configuring an array. The **Settings** window on the left is set to **Array** mode. The label is **Meander Line Center Conductor Middle Array**. Under **Input**, the **Input objects** list includes **r6**, **r7**, **r8**, and **r9**, with **r6** currently selected. The **Size** section shows **Array type** as **Rectangular**, **xw size** as **n_turns-1**, and **yw size** as **1**. The **Displacement** section has **xw** set to $2 * x_turns + 2 * w_cpw$ and **yw** set to **0**. The **Selections of Resulting Entities** section shows **Contribute to** as **None** and **Show in 3D** as **Domain selection**.

The **Graphics** window in the center shows a 2D plot of the meander line structure. The x-axis ranges from -0.004 to 0.003, and the y-axis ranges from -4.5 to 4.5. The structure consists of a central conductor with a meandered path, surrounded by a ground plane. The plot is scaled by $\times 10^{-2}$.

The **Physics** window on the right shows the **Physics interfaces in study** section with **Study 1** selected. The **Dependent Variables** section is empty.

The **Messages** window at the bottom shows the following text:
COMSOL Multiphysics 5.2.1.152
Finalized geometry is empty.
Opened file: cpw_meander.mph

1.17 GB | 6.05 GB

COMSOL RF Demo: Extrusions

The screenshot displays the COMSOL Multiphysics interface. On the left, the 'Settings' window for the 'Extrude' feature is open. The 'General' section shows 'Extrude from: Faces' and 'Input faces: wp1' with a list of faces 1, 3, and 5. The 'Distances from Plane' section has 'Distances (m): t_metal'. The 'Scales', 'Displacements', 'Twist Angles', and 'Polygon Resolution of Edges' sections are collapsed. The 'Selections of Resulting Entities' section shows 'Contribute to: None' and 'Resulting objects selection' set to 'Domain selection'. The 'Graphics' window shows a 3D model of a meander line structure on a rectangular substrate. A red arrow points upwards from the origin of the coordinate system, labeled 'Extrusion direction'. The 'Messages' window at the bottom shows the text: 'COMSOL Multiphysics 5.2.1.152', 'Finalized geometry is empty.', and 'Opened file: cpw_meander.mph'. The 'Physics' window on the right shows a list of physics interfaces, including 'Electromagnetic Waves, Frequency Domain', 'Magnetic Fields (mf)', 'Magnetic and Electric Fields', 'General Form Boundary PDE', 'Wave Equation (waeq)', 'AC/DC', 'Acoustics', 'Chemical Species Transport', 'Fluid Flow', 'Heat Transfer', 'Optics', 'Radio Frequency', and 'Structural Mechanics'. The 'Physics interfaces in study' section shows 'Study 1' and 'Solve'. The 'Dependent Variables' section is empty. The status bar at the bottom right shows '1.17 GB | 6.06 GB'.

COMSOL RF Demo: Meshing

The screenshot displays the COMSOL Multiphysics interface for a meander line structure. The **Model Builder** on the left shows the hierarchy: **cpw_meander_outfile_161221.mph (root)** > **Global Definitions** > **Component 1 (comp1)** > **Definitions** > **Geometry 1** > **Materials** > **Electromagnetic Waves, Frequency Domain (emw)** > **Mesh 1** > **Free Triangular 2**. The **Settings** panel for **Free Triangular 2** shows **Geometric entity level: Boundary** and **Selection: Manual**. A table lists the mesh elements:

Label	Count	Active
6	13	ON
21	25	
32	36	
294	299	

The **Graphics** window shows a 3D view of the meander line structure with a green mesh. The axes are labeled **x**, **y**, and **z**, with values ranging from -1×10^{-3} to 1×10^{-3} . The **Messages** window at the bottom displays the following information:

```
COMSOL Multiphysics 5.2.1.152
Opened file: cpw_meander_outfile_161221.mph
Mesh consists of 2376 boundary elements and 2162 edge elements.
Mesh consists of 30032 boundary elements and 5998 edge elements.
```

The status bar at the bottom right indicates **1.2 GB | 6.03 GB**.

COMSOL RF Demo: Meshing

The screenshot displays the COMSOL Multiphysics 5.2.1.152 interface during the meshing process. The main window shows a 3D perspective view of a rectangular domain with a fine tetrahedral mesh. The axes are labeled x, y, and z, with the x-axis ranging from -1×10^{-3} to 1×10^{-3} , the y-axis from -0.5×10^{-3} to 0.5×10^{-3} , and the z-axis from -4×10^{-3} to 0×10^{-3} .

The left panel shows the **Settings** for the **Free Tetrahedral** meshing method. The **Domain Selection** section is active, with the **Geometric entity level** set to **Remaining**. The **ON** button is highlighted, indicating that the meshing process is active. Other sections include **Scale Geometry**, **Control Entities**, **Tessellation**, and **Element Quality Optimization**.

The right panel shows the **Properties** section, which includes a list of **Recently Used** physics interfaces. The **Physics interfaces in study** section shows **Study 1** selected. The **Dependent Variables** section is currently empty.

The bottom panel shows the **Messages** window, which displays the following information:

```
COMSOL Multiphysics 5.2.1.152
Finalized geometry is empty.
Opened file: cpw_meander.mph
Mesh consists of 2376 boundary elements and 2162 edge elements.
Mesh consists of 30032 boundary elements and 5998 edge elements.
Complete mesh consists of 378490 domain elements, 70406 boundary elements, and 12408 edge elements.
```

The bottom right corner of the interface shows the memory usage: **1.16 GB | 13.48 GB**.

COMSOL RF Demo: Boundary Conditions, PEC

Settings
Perfect Electric Conductor
Label: Perfect Electric Conductor 2

Boundary Selection
Selection: Manual

ON

Active

- 4
- 15
- 23
- 303
- 306
- 308

Override and Contribution

Equation

Constraint Settings

Graphics

Messages Progress Log Table

COMSOL Multiphysics 5.2.1.152
Finalized geometry is empty.
Opened file: cpw_meander.mph

1.13 GB | 6.06 GB

Properties

Add Add

Add to Component

Add to Selection

Recently Used

- Electromagnetic Waves, Frequency Domain
- Magnetic Fields (mf)
- Magnetic and Electric Fields
- General Form Boundary PDE
- Wave Equation (waeq)

AC/DC

- Acoustics
- Chemical Species Transport
- Fluid Flow
- Heat Transfer
- Optics
- Radio Frequency
- Structural Mechanics
- Mathematics

Physics interfaces in study

Studies	Solve
Study 1	

Dependent Variables

COMSOL RF Demo: Boundary Conditions, PEC

Settings
Perfect Electric Conductor
Label: Perfect Electric Conductor 2

Boundary Selection
Selection: Manual

ON

Active

- 4
- 15
- 23
- 303
- 306
- 308

Override and Contribution
Equation
Constraint Settings

Graphics

PEC Boundaries

COMSOL Multiphysics 5.2.1.152
Finalized geometry is empty.
Opened file: cpw_meander.mph

1.13 GB | 6.06 GB

COMSOL RF Demo: Boundary Conditions, Ports

Terminal Settings →

Lumped Port 1

COMSOL Multiphysics 5.2.1.152
Finalized geometry is empty.
Opened file: cpw_meander.mph

1.12 GB | 6.06 GB

COMSOL RF Demo: Scattering Boundary Condition

The screenshot displays the COMSOL Multiphysics software interface. On the left, the 'Settings' window is open for a 'Scattering Boundary Condition'. The 'Boundary Selection' section shows a list of boundaries with '10' selected. The 'Equation' section is set to 'No incident field' and 'Plane wave'. The 'Scattered wave type' is set to 'First order'. The 'Graphics' window shows a 3D model of a meander structure with a green top surface and blue side surfaces. A black arrow points from the text 'Applied to all non-PEC and non-port exterior boundaries' to the top surface of the meander. The 'Messages' window at the bottom shows the text: 'COMSOL Multiphysics 5.2.1.152', 'Finalized geometry is empty.', and 'Opened file: cpw_meander.mph'. The 'Physics interfaces in study' window on the right shows 'Radio Frequency' selected.

Applied to all non-PEC and non-port exterior boundaries

COMSOL Multiphysics 5.2.1.152
Finalized geometry is empty.
Opened file: cpw_meander.mph

1.1 GB | 6.06 GB

COMSOL RF Demo: Frequency Sweep

Settings

Frequency Domain

Compute Update Solution

Label: Frequency Domain

Study Settings

Frequency unit: Hz

Frequencies: range(6[GHz], 0.0125[GHz], 7[GHz]) Hz

Load parameter values: Browse... Read File

Reuse solution from previous step: Auto

Results While Solving

Physics and Variables Selection

Modify physics tree and variables for study step

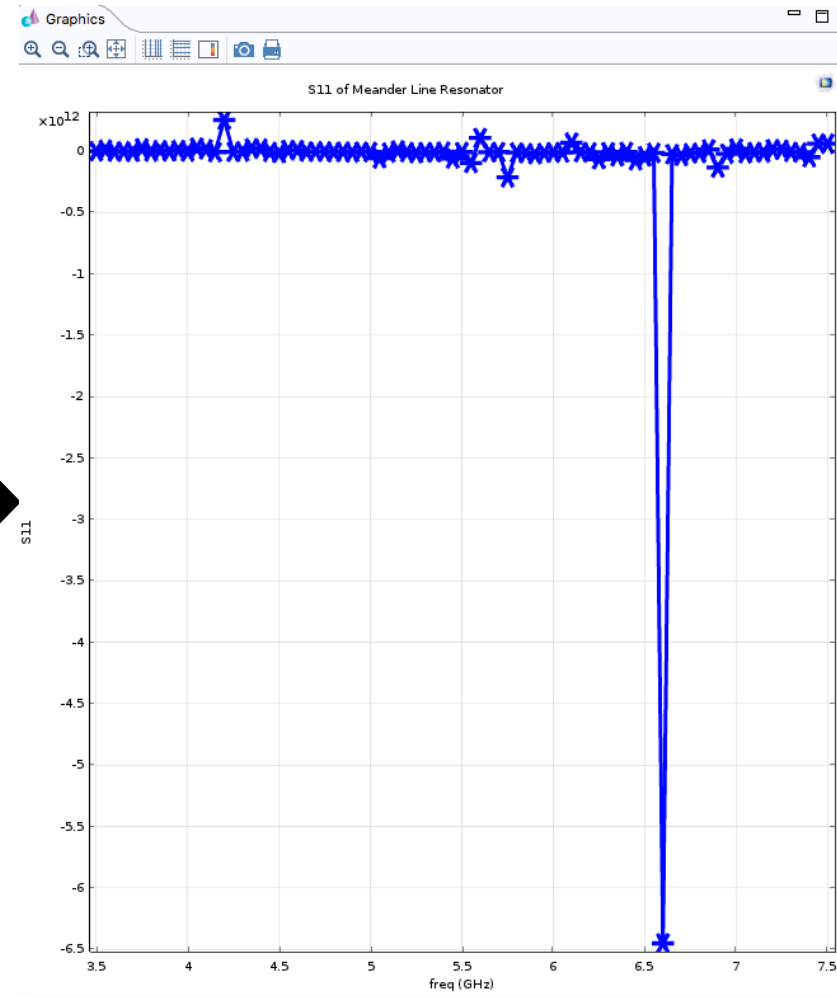
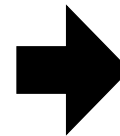
Physics interface	Solve for	Discretization
Electromagnetic Waves, Frequency...	<input checked="" type="checkbox"/>	Physics settings

Multiphysics Solve for

Values of Dependent Variables

Mesh Selection

Study Extensions



COMSOL RF Demo: E-Field Plot

The screenshot displays the COMSOL Multiphysics interface for a meanderline structure simulation. The main plot shows the electric field norm (V/m) in a 3D perspective view. The structure is a meanderline on a rectangular substrate, with a central gap. The plot is titled "freq(62)=6.55E9 Hz Multislice: Electric field norm (V/m)". A color bar on the right indicates the field strength, ranging from 0 (blue) to 7 (red) $\times 10^3$ V/m. The plot is set to "Multislice" mode.

Settings Panel (Multislice):

- Data:** Data set: Study 1/Solution 1 (sol1); Parameter value (freq (Hz)): 6.55E9
- Expression:** Expression: emw.normE; Unit: V/m
- Title:** Description: Electric field norm
- Multiplane Data:**
 - x-planes: Entry method: Number of planes; Planes: 1
 - y-planes: Entry method: Number of planes; Planes: 0
 - z-planes: Entry method: Coordinates; Coordinates: 0 m
- Range:** Coloring and Style: Color table; Color table: Rainbow; Color legend: ; Reverse color table: ; Symmetrize color range:
- Quality:** **Inherit Style**

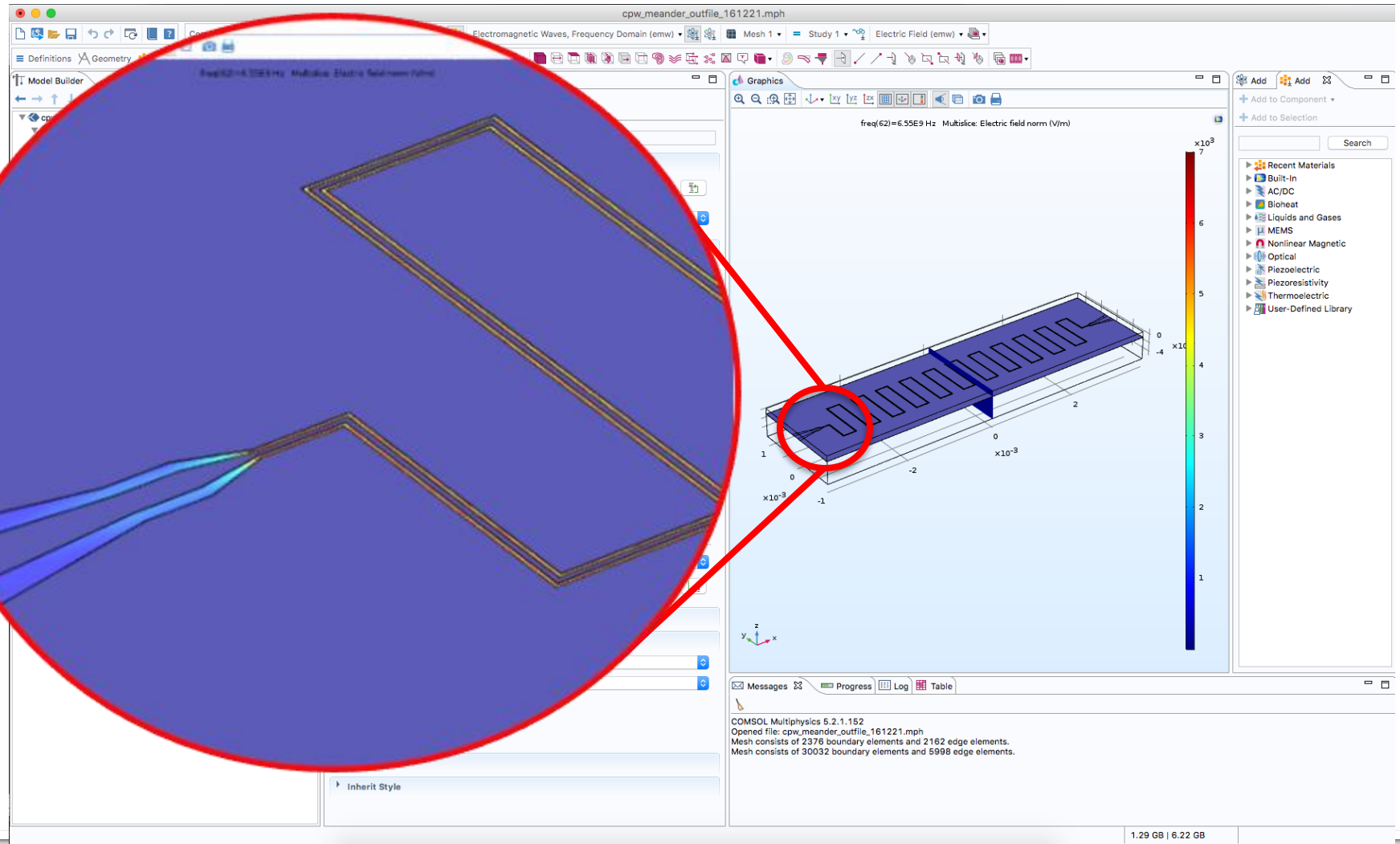
Graphics Panel: Messages: Progress; Log; Table

Model Builder: cpw_meander_outfile_161221.mph (root) > Global Definitions > Component 1 (comp1) > Electromagnetic Waves, Frequency Domain (emw) > Mesh 1 > Study 1 > Results > Electric Field (emw) > Multislice > 1D Plot Group 3 > S11 from Voltage and Current

Material Library: Recent Materials, Built-In, AC/DC, Bioheat, Liquids and Gases, MEMS, Nonlinear Magnetic, Optical, Piezoelectric, Piezoresistivity, Thermoelectric, User-Defined Library

COMSOL Multiphysics 5.2.1.152
Opened file: cpw_meander_outfile_161221.mph
Mesh consists of 2376 boundary elements and 2162 edge elements.
Mesh consists of 30032 boundary elements and 5998 edge elements.

COMSOL RF Demo: E-Field Plot



Closing Comments on cQED and COMSOL

- Superconducting qubits benefit from simple descriptions in cQED by through analogies with cavity QED
- Black box quantization provides a systematic method of quantizing the bulk features of devices as circuits
- COMSOL provides a simulation environment to model the classical geometric features of qubits

Acknowledgements

- Jonathan DuBois, Eric Holland, Matthew Horsley, Vincenzo Lordi, Scott Nelson, Yaniv Rosen, Nathan Woollett
- This work was funded by the LLNL Laboratory Directed Research and Development (LDRD) program, project number 15-ERD-051.



Thank you—Questions



