Simulation of Superconducting Qubits Using COMSOL

Workshop on Microwave Cavity Design
for Axion Detection

Nick Materise

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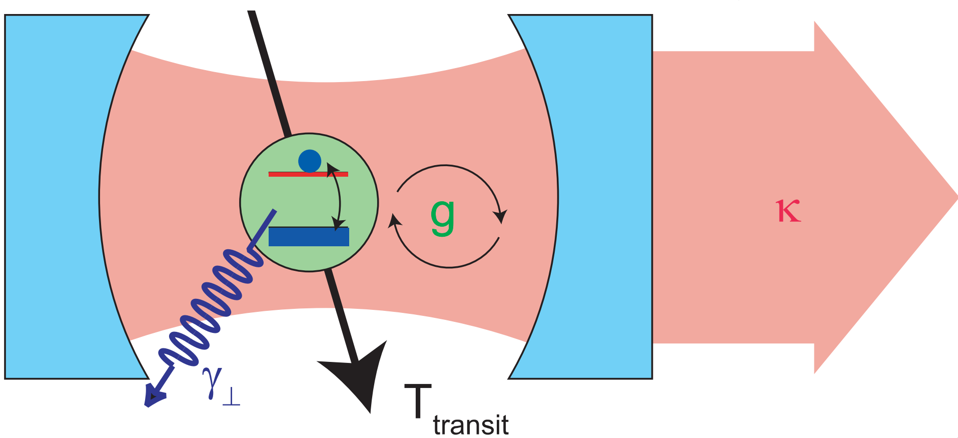
Outline

- Review of Cavity QED and Circuit QED
- COMSOL RF Modeling
- Noise Parametrization in COMSOL
- Connections to ADMX Models
- Summary
- Future Work

Review of Cavity QED Systems

- Cavity QED: two level atomic system trapped in a mirrored resonant cavity
- Follows the Jaynes-Cummings Hamiltonian[1]:

$$\hat{H} = \underbrace{\hbar\omega_r(\hat{a}^\dagger\hat{a} + 1/2)}_{\text{EM field quantization}} + \underbrace{\frac{1}{2}\hbar\omega_a\hat{\sigma}_z}_{\text{spin}-1/2\text{ atom}} + \underbrace{\hbar g(\hat{a}^\dagger\hat{\sigma}_- + \hat{a}\hat{\sigma}_+)}_{\text{interaction between atom, cavity}} \quad (1)$$



Atom trapped in a cavity with photon emission, atomic-cavity dipole coupling, and atom transit time shown[2].

$2g$ = Vacuum Rabi Frequency
κ = Cavity Decay Rate
γ = Transverse Decay Rate
T = Time for atom to leave cavity

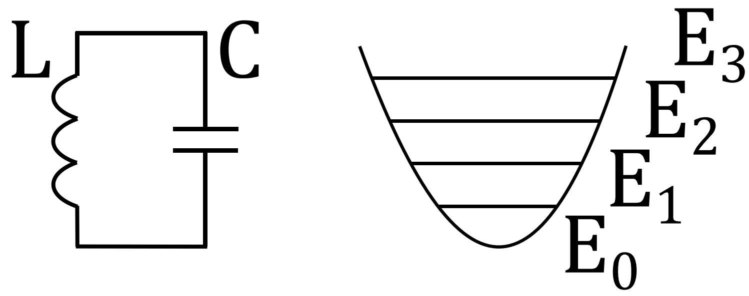
[1] D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

[2] R. J. Schoelkopf and S. M. Girvin, "Wiring up quantum systems," Nature, vol. 451, pp. 664–669, 02 2008.

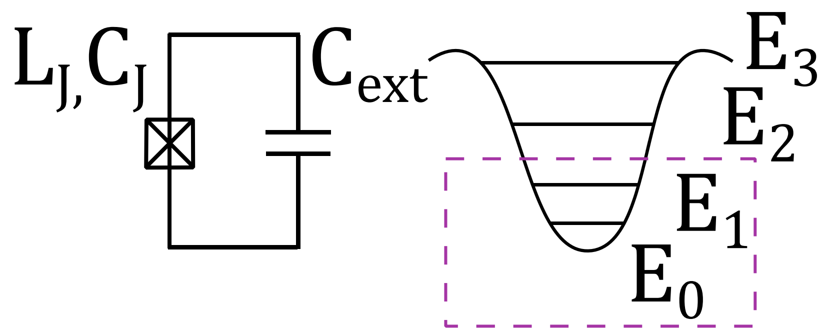
Superconducting Qubits and Circuit QED

- Circuit QED ~ Cavity QED
  - Harmonic Oscillator as a first circuit model approximation[1]
  - Add non-linear inductance with Josephson Junction, anharmonicity

- "Artificial Atom" or qubit replaces atom from Cavity QED
  - Two lowest energy levels form a two level system (TLS)
  - Qubit = TLS formed by Josephson Junction and other circuit elements[3]



LC harmonic oscillator circuit



TLS, anharmonic oscillator

[1] D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

[3] William D. Oliver and Paul B. Welander. Materials in superconducting quantum bits. MRS Bulletin, 38:816–825, 10 2013.

Cavity QED—Circuit QED Comparison

Parameter	Symbol	Cavity QED[1]	Circuit QED[1,4]
Resonator, Qubit Frequencies	$\omega_r, \omega_q / 2\pi$	~ 50 GHz	~ 5 GHz
Transition Dipole Moment	d / ea_0	~ 1	~ 10^4
Relaxation Time	T_1	30ms	60 μs
Decoherence Time	T_2	~1 ms	~10-20 μs

- Large dipole moment couples the qubit well to the cavity in superconducting qubits: coupling strength and energy levels are *tunable*
- Trapped atoms in cavities have *longer coherence times*, *not tunable*, weakly coupled to the cavity for measurement

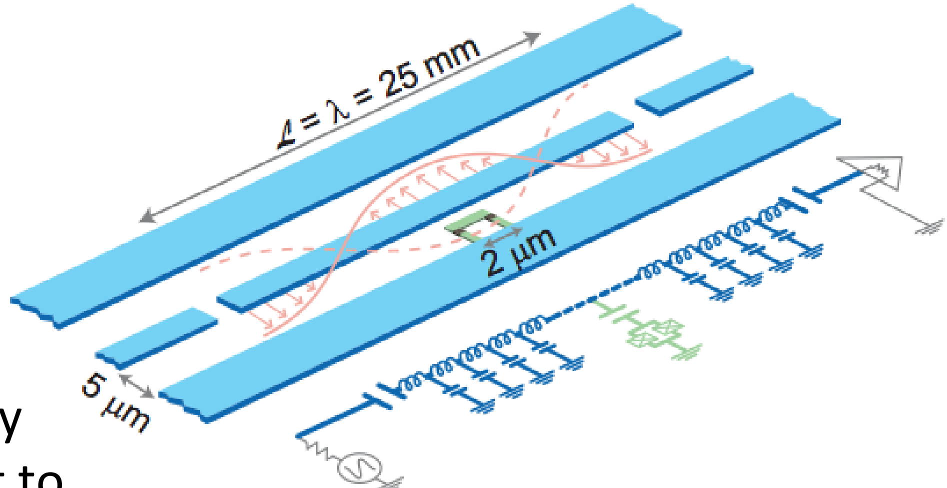
[1] D. I. Schuster, Circuit Quantum Electrodynamics. PhD thesis, Yale University, 2007.

[4] Hanhee Paik, D. I. Schuster, Lev S. Bishop, G. Kirchmair, G. Catelani, A. P. Sears, B. R. Johnson, M. J. Reagor, L. Frunzio, L. I. Glazman, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf. Observation of High Coherence in Josephson Junction Qubits Measured in a Three-Dimensional Circuit QED Architecture. Phys. Rev. Lett., 107:240501, Dec 2011.

Superconducting Qubit Design

- Implemented as Josephson Junction and large capacitance coupled to transmission line resonator (coplanar waveguide, CPW)
  - Transmission Line Resonator ~ Cavity
  - 2D Planar or 3D cavity couples qubit to readout hardware
- Dipole Moment, resonator modes, energy levels—all tunable in microwave and Josephson Junction design

[5] Alexandre Blais, Ren-Shou Huang, Andreas Wallraff, S. M. Girvin, and R. J. Schoelkopf. Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation. Phys. Rev. A, 69:062320, Jun 2004.



Coplanar waveguide resonator and lumped circuit element reproduced from[5].

Dipole Moment (d)

$$\hbar g = (ew)\frac{1}{w}V_0 = dE_0,$$

where w is the width of the center conductor in the CPW

Superconducting Qubit Design—COMSOL Modeling

- Key advantage of Circuit QED is its tunability through fabrication
- Emphasis on design microwave circuitry calls for detailed classical and quantum models of noise
- COMSOL and other modeling tools provide a means to parameterize noise sources

Develop a simple geometry for design under test

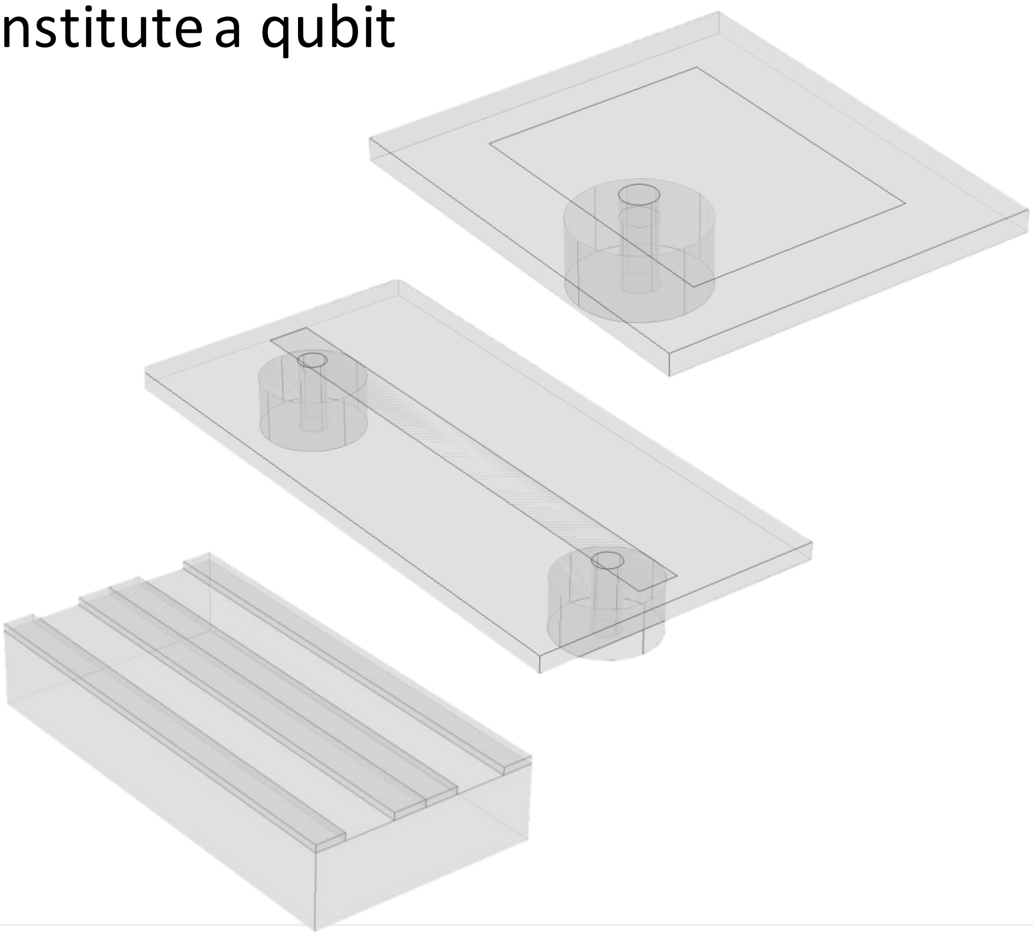
Identify materials properties; apply them to the model

Select an excitation method; apply boundary conditions (Dirchlet, Neumann, Robin, PEC)

COMSOL RF Simulation Studies

- Model systems used to develop more accurate descriptions of the microwave circuits that constitute a qubit
- Model Progression

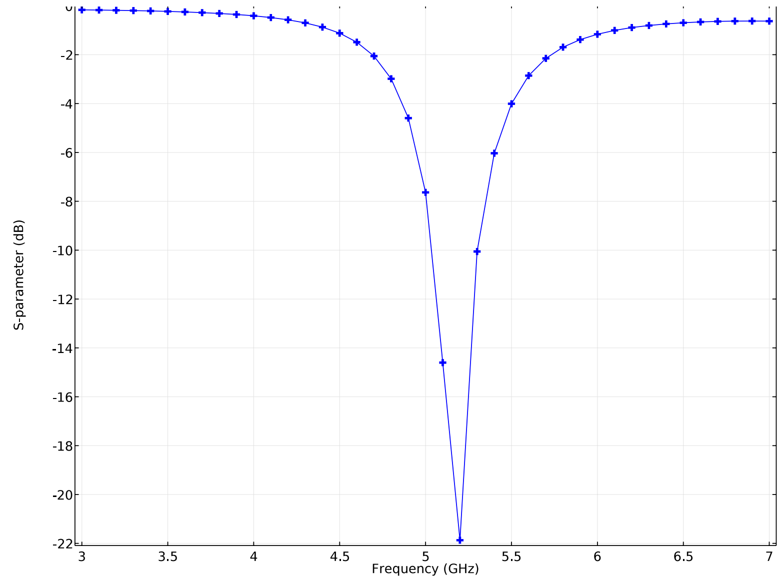
1. Microstrip Patch Antenna (MPA)
2. Microstripline Antenna
3. Coplanar Waveguide (CPW)

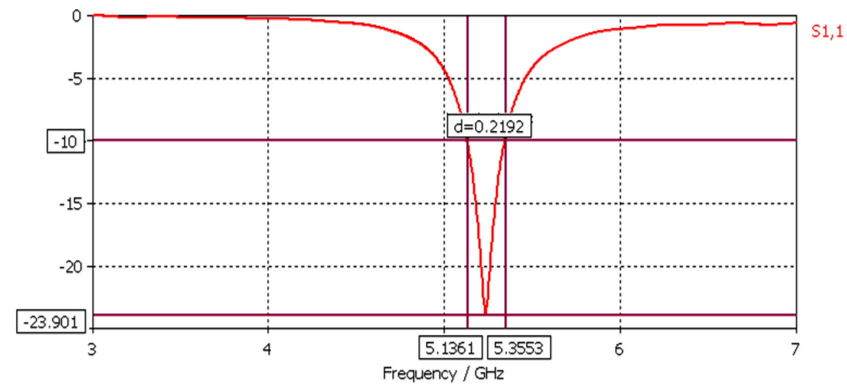


Microstrip Patch Antenna

- Simple geometry, application in consumer antenna designs, military devices, and other areas where space is premium

- We used a simple coaxial excitation to reproduce the return loss results from[6] to verify our COMSOL model





[6] J. Kaur and R. Khanna. Co-axial Fed Rectangular Microstrip Patch Antenna for 5.2 GHz WLAN Application. Universal Journal of Electrical and Electronic Engineering, 1(3):94–98, 2013.

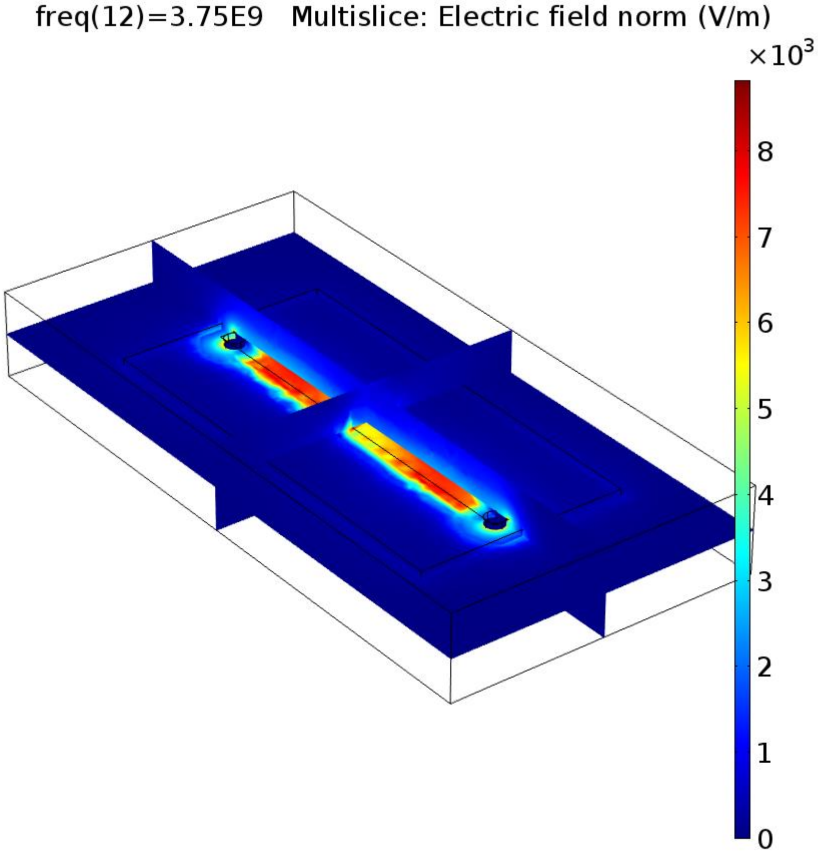
Microstripline Resonator

- Two port extension of the MPA used to *probe* a substrate by measuring S_{21}
- Characteristic Impedance given by:

$$Z_0 = \frac{87}{\sqrt{\varepsilon_r + 1.41}} \ln\left(\frac{5.98h}{0.8w + t}\right)$$

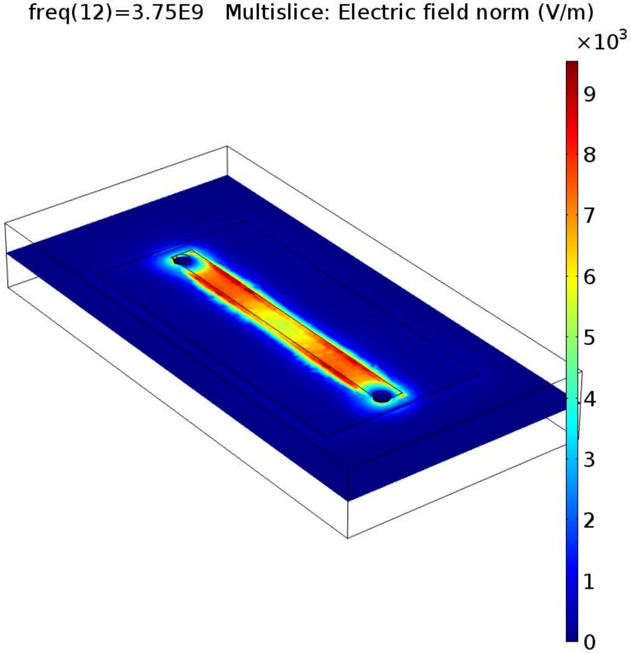
where h is the substrate thickness,
t the thickness of the microstrip,
w the width of the strip.

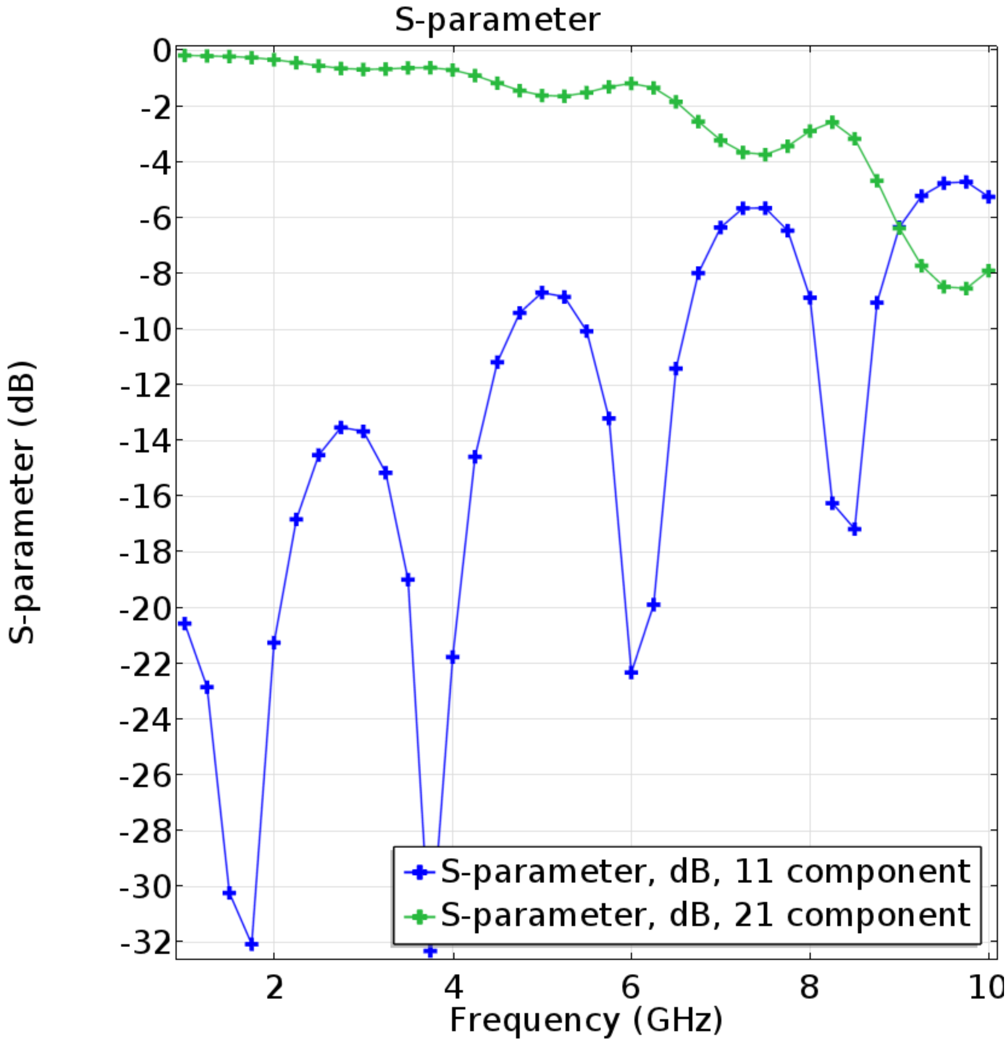
- Electric field plot at resonance



Microstripline Resonator

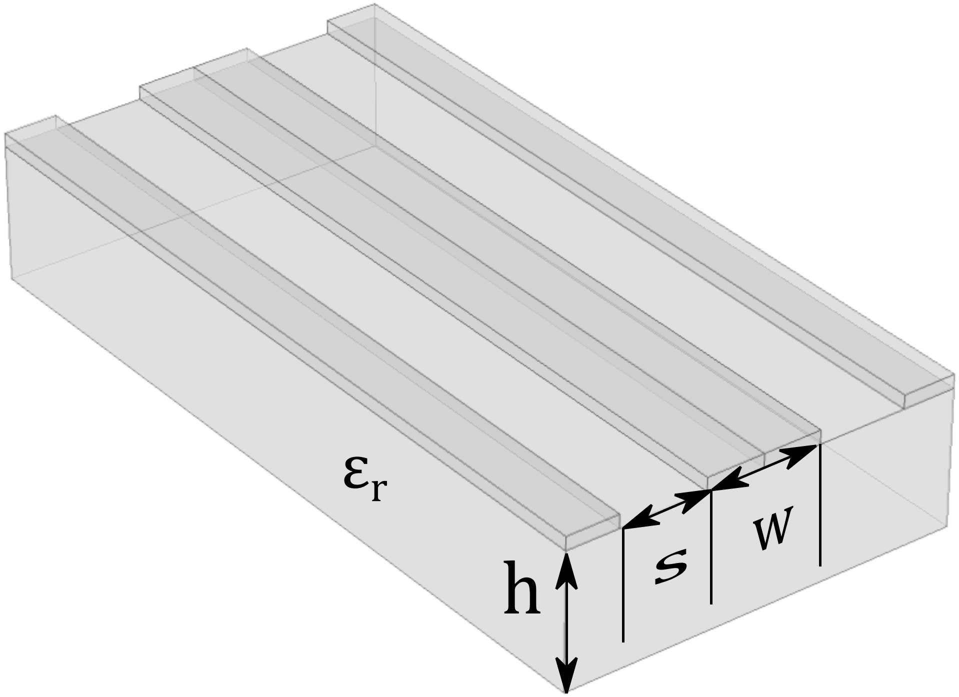
- S_{11}, S_{21} measured for PTFE substrate with dielectric constant $\varepsilon_r = 2.5$ and loss tangent $\tan\delta = 0.0009$





Coplanar Waveguide

- Coplanar waveguide used as a resonant coupling structure, i.e. *cavity* with the qubit



- Characteristic Impedance from conformal mapping[7]:

$$Z_0 = \frac{30\pi}{\sqrt{\varepsilon_{\text{eff}}}} \frac{K(k_0')}{K(k_0)}$$

where,

$$\varepsilon_{\text{eff}} = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k_1)}{K(k_1')} \frac{K(k_0')}{K(k_0)}$$

$$k_0 = \frac{s}{s + 2w}$$

$$k_0' = \sqrt{1 - k_0^2}$$

$$k_1 = \frac{\sinh(\pi s/4h)}{\sinh(\pi(s + 2w)/4h)}$$

$$k_1' = \sqrt{1 - k_1^2}$$

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}}$$

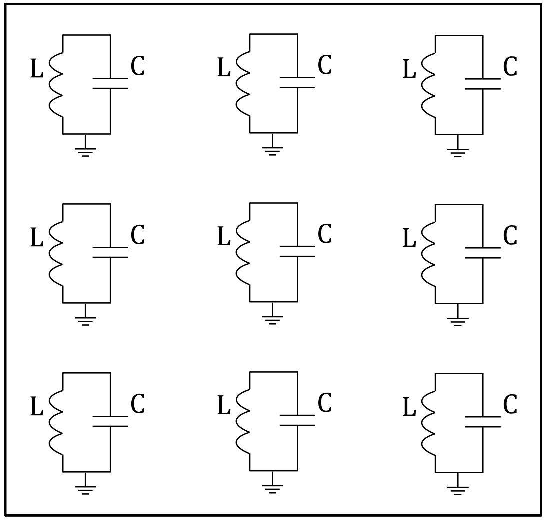
[7] Rainee N Simons. Coplanar Waveguide Circuits, Components, and Systems, chapter 2. Wiley Series in Microwave and Optical Engineering. Wiley, 2001.

Noise Parametrization

- Goal: develop parametrized models of *microscopic noise* that are measurable in *macroscopic observables*
- Two approaches to injecting noise in our COMSOL models:

1. Discrete Noise Sources
   a) **Lumped circuit elements**
   b) Point defects
   c) Other discrete atomic / mesoscopic imperfections

1. Continuous Noise Sources
   a) **Analytical models of material properties, i.e. $\varepsilon_r(x,y,z,t), \mu_r(x,y,z,t), \sigma(x,y,z,t)$**
   b) Noise power spectral density function $S(f)$



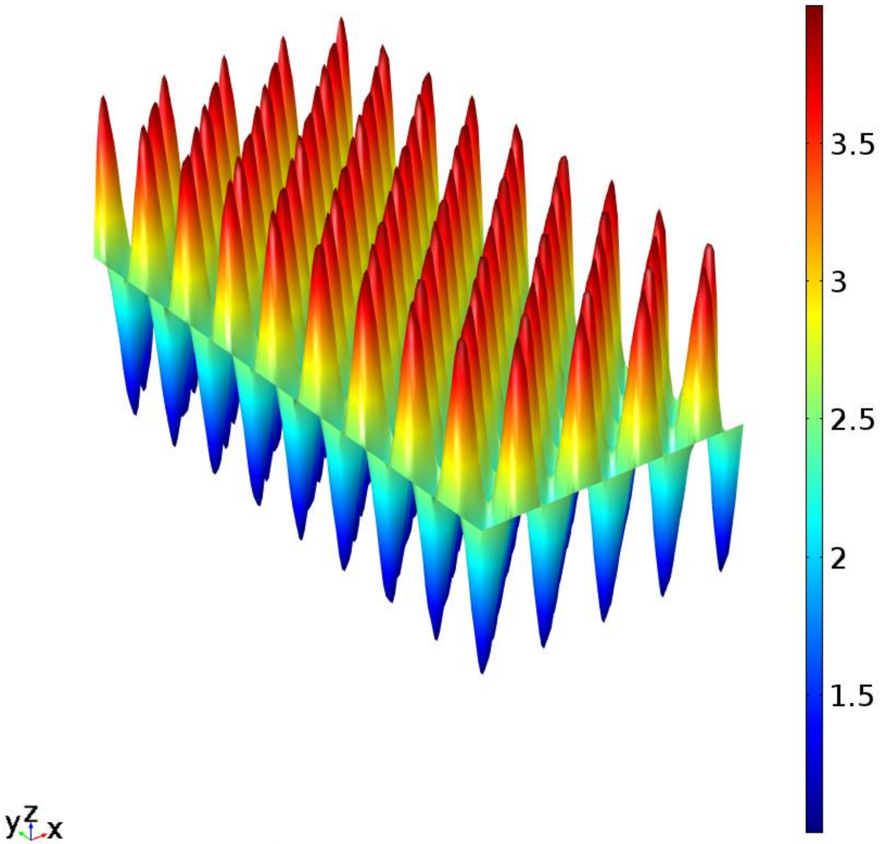
Lumped circuit model of discrete LC-oscillators dispersed throughout the bulk of a material / substrate to model discrete harmonic oscillator noise.

Spatially Varying Permittivity

- Noise injected into COMSOL models via bulk parameters
- Permittivity expressed as a function of space (time):

$$\varepsilon_r = f(x, y, z, t)$$
$$\varepsilon_r(x, y) = A\, \varepsilon_{r_0} \sin(w_x x) \sin(w_y y)$$

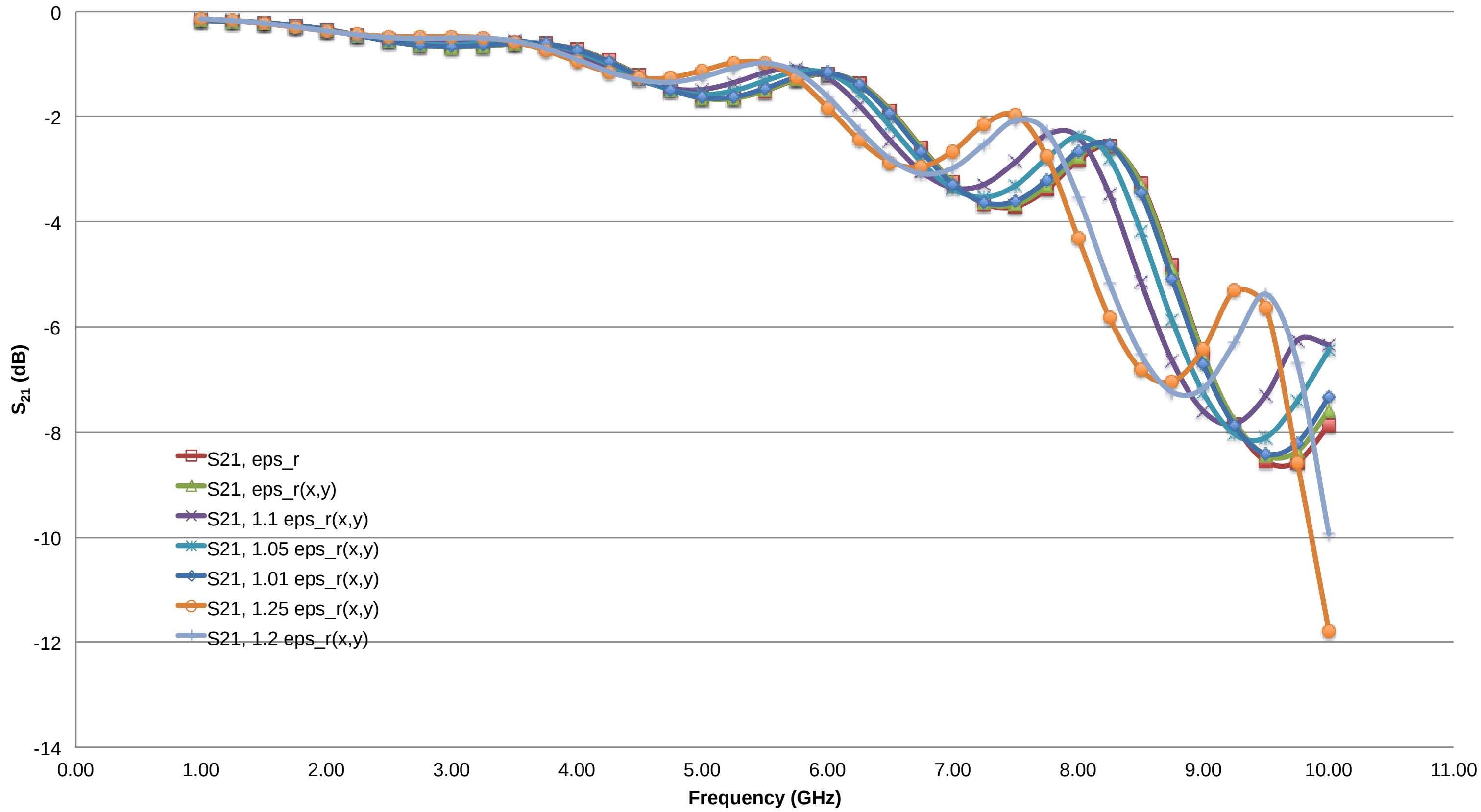
with ε_{r_0}, the original permittivity constant A, and w_x, w_y, spatial frequencies



Surface plot of spatially varying permittivity as an analytic function in COMSOL. Color intensity reflects the relative magnitude of the function in the z-direction.

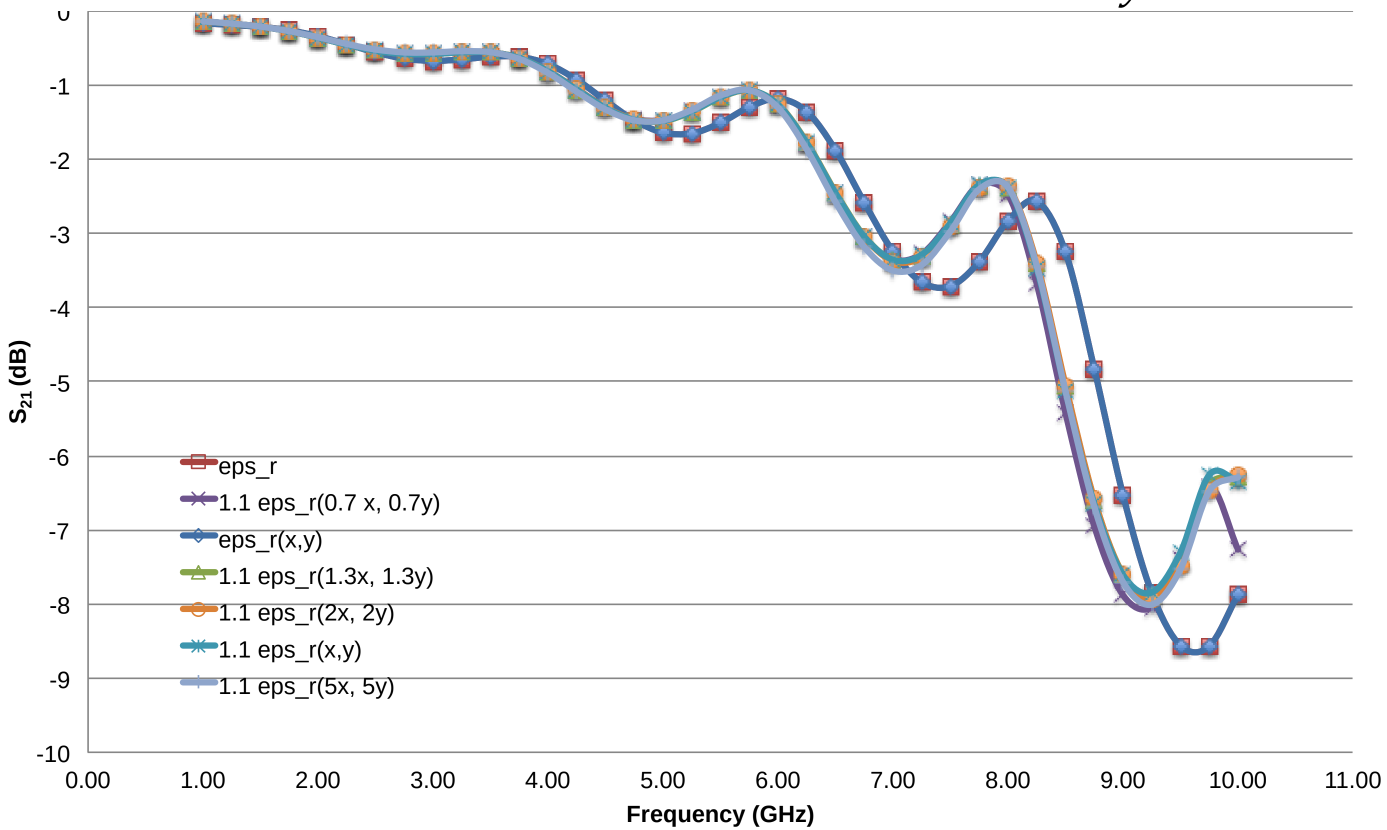
Spatially Varying Permittivity COMSOL Study, 1

- Microstripline resonator used as the model system
- Progression of S_{21} for different scaling values, A



Spatially Varying Permittivity COMSOL Study, 2

- Microstripline resonator used as the model system
- Progression of S_{21} for different frequencies, w_x, w_y



COMSOL Modeling—ADMX Contributions

- Microwave cavity simulation: COMSOL Blog post by Bjorn Sjodin

Summary

- Circuit QED allows us to emulate Cavity QED systems in solid state devices, affording tunability and exploring new physics on chip
- Qubits in the Circuit QED architecture require microwave engineering and involve microscopic noise sources
- COMSOL allows for rapid design of RF circuits and flexible modeling of underlying physics in materials
- Noise sources are conveniently modeled by *discrete* or *continuous* distributions
- COMSOL models provide a means to test macroscopic manifestations of microscopic noise in subsequent experiments

Future Work

- Continue to develop more descriptive COMSOL models that incorporate bulk parameters with various dependencies
- Simulate designs that more closely resemble those in existing superconducting qubits
- Develop methods of modeling dissipation to predict relaxation and coherence times
- Design and run experiment (s) to validate more comprehensive models

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