Quantum heat engine simulated on superconducting qubits

Nick Materise and Eliot Kapit

[1]Colorado School of Mines, Department of Physics

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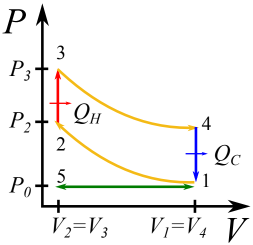
Classical Otto Engine

1. Intake stroke
2. Compression stroke (adiabatic), $V_1 \to V_2$, $P_0 \to P_2$
3. Heating stroke (isochoric), $P_2 \to P_3$
4. Expansion stroke (adiabatic), $V_2 \to V_1$, $P_3 \to P_2$
5. Cooling stroke (isochoric), $P_3 \to P_0$

≫ Efficiency $\eta_{\text{Otto}} = 1 - r^{1-\gamma}$, $r = V_1/V_2$, $\gamma = C_v/C_p$ [1]

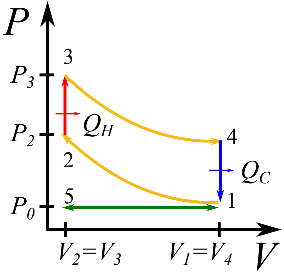


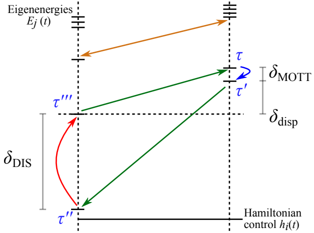
[1] Quattroch 2006.

Near Ground State Disordered (NGSD) Engine

1. $\tau''' \rightarrow \tau$, Expansion
2. $\tau \rightarrow \tau'$, Cooling
3. $\tau' \rightarrow \tau''$, Compression
4. $\tau'' \rightarrow \tau'''$, Heating

» Efficiency
$\eta_{\mathrm{NGSD}} = 1 - \delta_{\mathrm{MOTT}}/\delta_{\mathrm{DIS}}$, similar to[2]

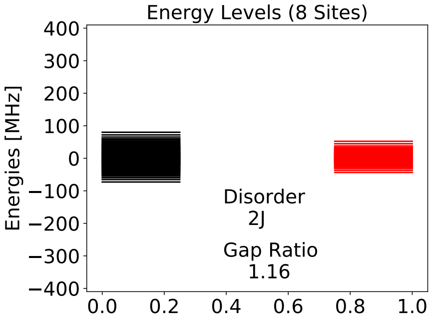




[2] Yunger Halpern et al., Phys. Rev. B **99**, 024203 (2019).

Near Ground State Disordered (NGSD) Engine

- Energy Spectrum of disordered (DIS) and Mott (MOTT) insulator phases



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Near Ground State Disordered (NGSD) Engine

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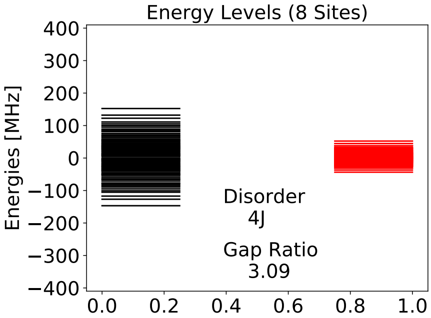
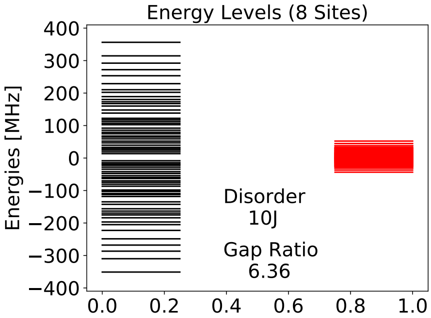
- Compared to the MBL engine, the NGSD engine does *not* operate at the middle of the spectrum



[2] Yunger Halpern et al., Phys. Rev. B **99**, 024203 (2019).

Near Ground State Disordered (NGSD) Engine

- Energy Spectrum of disordered (DIS) and Mott (MOTT) insulator phases
- Compared to the MBL engine, the NGSD engine does *not* operate at the middle of the spectrum
- This allows us to achieve high efficiencies $(1 - 1/\text{Gap Ratio})$ in shorter times



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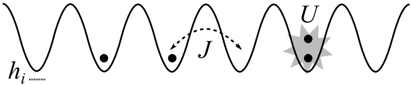
Bose Hubbard Model (Standard)



- Hamiltonian[3]

$$H = -\sum_{\langle ij \rangle} J_{ij}(a_i^\dagger a_j + \text{h.c.}) + \frac{U}{2}\sum_i n_i(n_i+1) + \sum_i (h_i - \mu)\, n_i \qquad (1)$$

[3] Ma et al., Nature **566**, 51 (2019).

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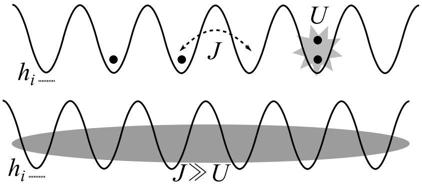
- Superfluid phase



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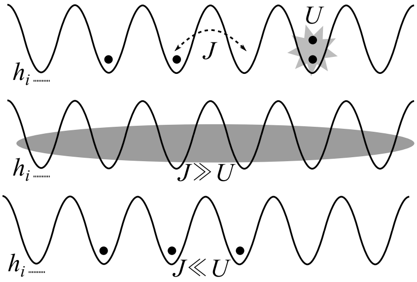
- Superfluid phase
- Mott insulator phase (MOTT)



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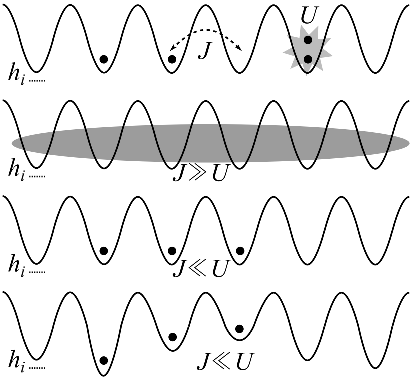
- Superfluid phase
- Mott insulator phase (MOTT)
- Disordered phase (DIS)



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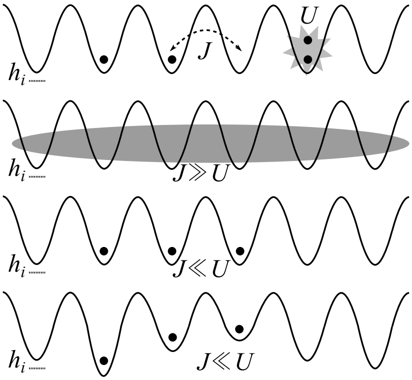

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Quantum Annealing

- Adiabatic evolution of $H_0 \rightarrow H_p$[4]

$$H(t) = H_0 A(t) + H_p B(t) \qquad (2)$$

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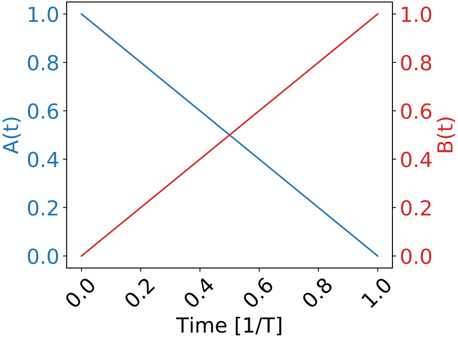

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Quantum Annealing

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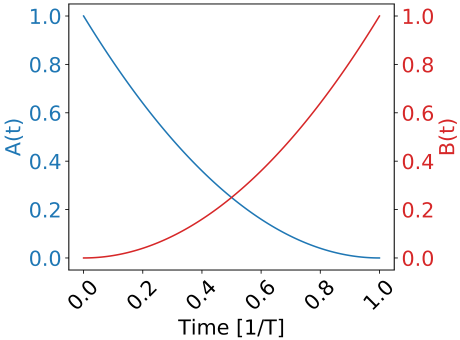
$$H(t) = H_0 A(t) + H_p B(t) \quad (2)$$

- Bose Hubbard as annealing problem

$$H(t) = H_0 + C(t) \sum_i h_i n_i, \quad (3)$$

$$C(t) = \begin{cases} A(t), & \text{DIS} \rightarrow \text{MOTT} \\ B(t), & \text{MOTT} \rightarrow \text{DIS} \end{cases} \quad (4)$$



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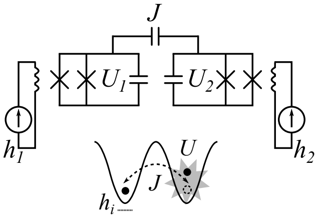
Proposed Experimental Realization

1. Superconducting circuit elements[3]



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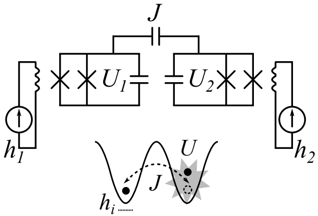

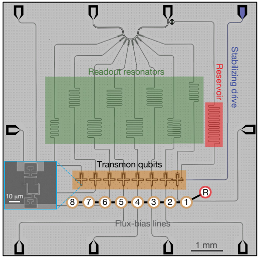

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Proposed Experimental Realization

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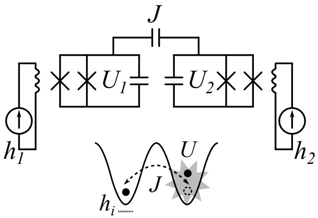
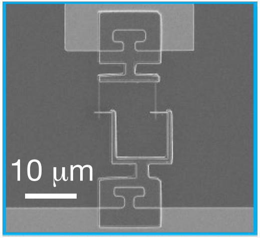
2. Device layout
3. SQUIDs forming tunable transmons





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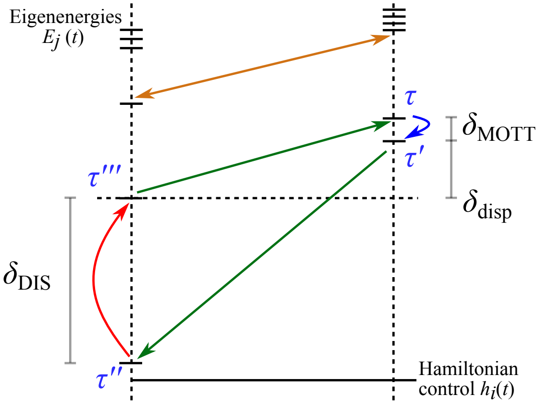
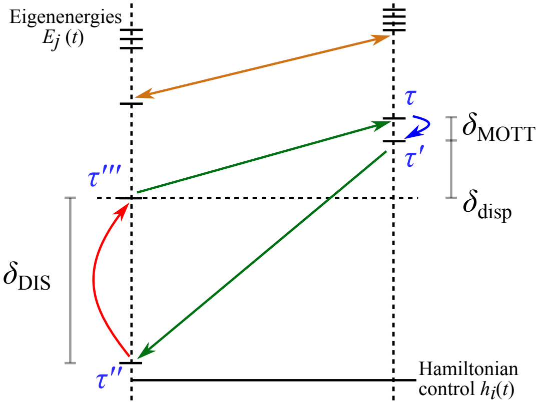
Results – *Compression & Expansion* Strokes

1. Evolve adiabatically between MOTT & DIS eigenstates, $(\tau' \rightarrow \tau'')$, $(\tau''' \rightarrow \tau)$



Results – *Compression & Expansion* Strokes

1. Evolve adiabatically between MOTT & DIS eigenstates, $(\tau' \to \tau'')$, $(\tau''' \to \tau)$
2. Calculate fidelity $|\langle\psi_f|\psi_t\rangle|^2$ and residual energy $|E_t - \langle H\rangle|$



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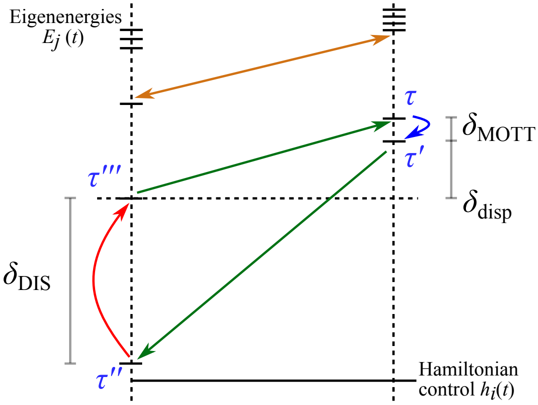
3. Repeat, average over 100 different $h_i \in [-10J,\ 10J]$ realizations



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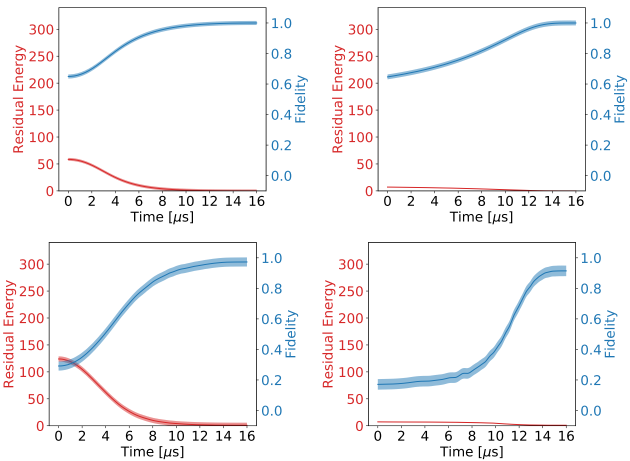
4. 2,4 sites

Compression $(\tau' \to \tau'')$ **Expansion** $(\tau''' \to \tau)$



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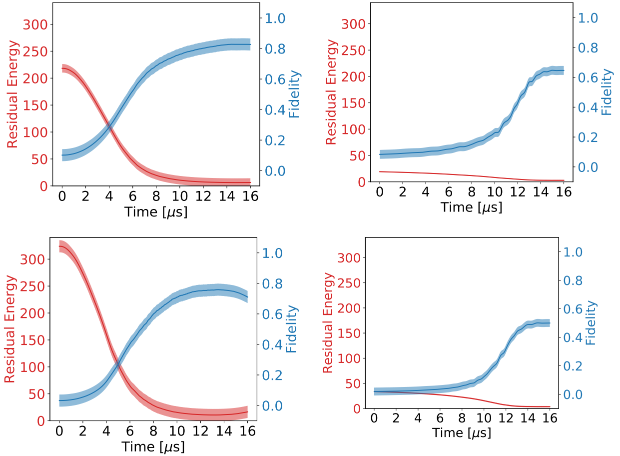
4. 6,8 sites

Compression $(\tau' \to \tau'')$ **Expansion** $(\tau''' \to \tau)$



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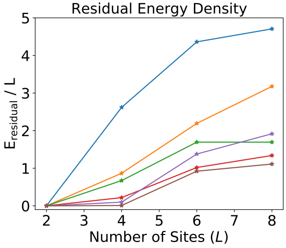
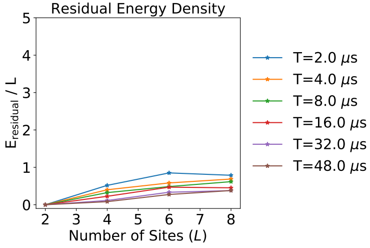
4. Residual Energy Density

Compression $(\tau' \to \tau'')$

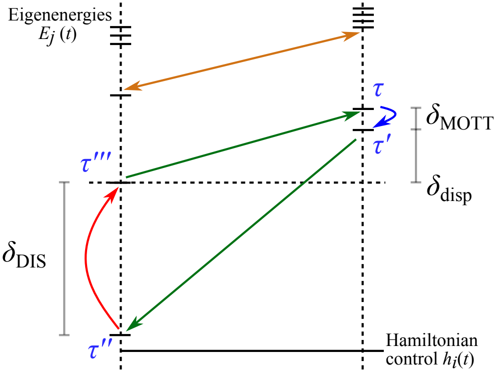


Expansion $(\tau''' \to \tau)$



Outlook and Conclusions

- NGSD engine projected to have efficiency as function of ratio of MOTT / DIS gaps



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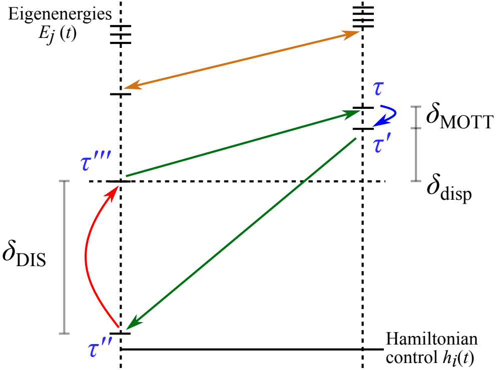

Outlook and Conclusions

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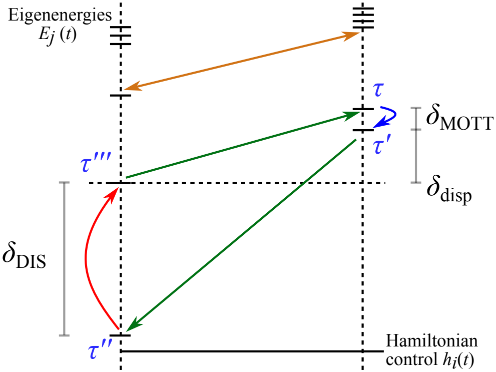
- Repeat calculations on the DWAVE machine, explicit modeling of the cold bath



Acknowledgements

- Kapit Group: Eric Jones, David Rodríguez Pérez, Zhijie Tang, Mallory Zabrocky

  F07.00006 Variational Preparation of Quantum Hall States on a Lattice

  L16.00004 Is Fault Welcoming Quantum Computing Realistic?

  L16.00010 Oscillatory quantum optimization methods applied to problems with large ground state bands

  M09.00005 Considerations for incorporating small logical qubits in digital error correction codes
- Graduate Fellowship for Science, Technology, Engineering, and Mathematics Diversity (formerly, National Physical Sciences Consortium Fellowship)
- NSF Grant PHY-1653820

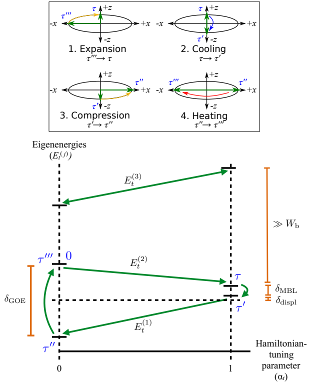
Additional Slides

Many Body Localized (MBL) Engine

- Single qubit engine cycle[3]

$$H(t) = (1-\alpha_t)\frac{\delta_{\text{GOE}}}{2}\sigma^x + \alpha_t \frac{\delta_{\text{MBL}}}{2}\sigma^z \quad (5)$$

1. $\tau''' \to \tau$, Expansion
2. $\tau \to \tau'$, Cooling
3. $\tau' \to \tau''$, Compression
4. $\tau'' \to \tau'''$, Heating



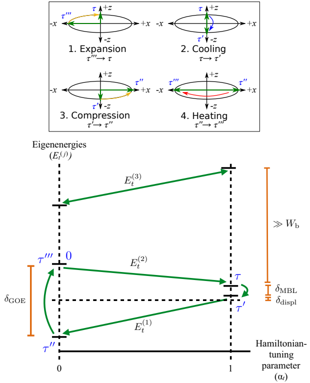
[3] Yunger Halpern et al., Phys. Rev. B **99**, 024203 (2019).

Many Body Localized (MBL) Engine

- Heat and work absorbed[3]

$$Q := \int_0^\tau \mathrm{Tr}\left(\rho \frac{dH}{dt}\right) dt \qquad (6)$$

$$W := \int_0^\tau \mathrm{Tr}\left(\frac{d\rho}{dt} H\right) dt \qquad (7)$$



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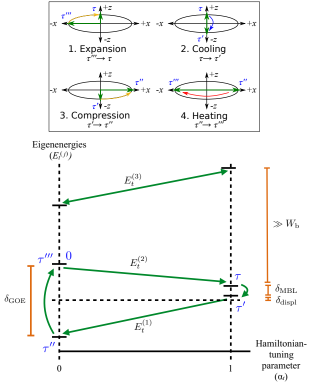
Many Body Localized (MBL) Engine

- Applied to qubit MBL engine[3]

$$\langle Q_{\text{in}} \rangle = \langle Q_4 \rangle = \frac{\delta_{\text{GOE}}}{2} \tag{5}$$

$$\langle W_{\text{tot}} \rangle = \frac{\delta_{\text{GOE}}}{2} + \frac{\delta_{\text{MBL}}}{2} \tag{6}$$

$$\eta_{\text{MBL}} = \frac{\langle W_{\text{tot}} \rangle}{\langle Q_{\text{in}} \rangle} = 1 - \frac{\delta_{\text{GOE}}}{\delta_{\text{MBL}}} \tag{7}$$



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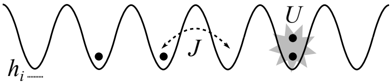
Bose Hubbard Model (Two-Level Perturbative)



- Hamiltonian

$$H = -J \sum_{i=1}^{N_s-1} \left(\sigma_i^+ \sigma_{i+1}^- + \text{h.c.}\right) + \frac{4J}{U^2} \sum_{i=1}^{N_s-1} \frac{1}{2}\left(\mathbb{1} + \sigma_i^z\right) \frac{1}{2}\left(\mathbb{1} + \sigma_{i+1}^z\right) + \frac{2J}{U^2} \sum_{i=1}^{N_s-2} \left(\sigma_{i+2}^+ \sigma_{i+1}^+ \sigma_{i+1}^- \sigma_i^- + \text{h.c.}\right) \quad (8)$$

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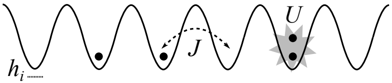

- Note, all operators are now particle number conserving

$$\sigma_i^+ \sigma_j^- |ij\rangle = |ji\rangle \quad (9)$$

$$\sigma_i^z |ij\rangle = \text{sign}(i-1)\,|ij\rangle \quad (10)$$

References

Farhi, Edward et al. (2000). *Quantum Computation by Adiabatic Evolution*. arXiv: quant-ph/0001106 [quant-ph].

Ma, Ruichao et al. (2019). "A dissipatively stabilized Mott insulator of photons". In: *Nature* 566.7742, 51. DOI: 10.1038/s41586-019-0897-9.

Quattroch, D. (2006). *The internal combustion engine (Otto cycle)*. http://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node26.html.

Yunger Halpern, Nicole et al. (Jan. 2019). "Quantum engine based on many-body localization". In: *Phys. Rev. B* 99 (2), 024203. DOI: 10.1103/PhysRevB.99.024203.